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THE MATHEMATICS TEACHER

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VOLUME XXII **NOVEMBER, 1929** NUMBER 7

SIGNIFICANT FACTS IN THE HISTORY OF THE METRIC SYSTEM FOR TEACHERS OF JUNIOR AND SENIOR HIGH SCHOOL MATHEMATICS

BY EDWIN W. SCHREIBER

Western Illinois State Teachers College, Macomb, Ill.

It was down in old Mexico that I had my first *real* introduction to the metric system. 'Tis true I had met the meter and her children, centimeter, millimeter, and little micron, in a formal way while busy in the laboratories at the university in my undergraduate days—but it was a cold and scientific acquaintance. Under a warm southern sky, with the sun doing its full share to brighten the picture, I read in no uncertain letters on a freshly painted sign which was posted on a little railway station in old Sonora: "To Calexico, 33.5 Km." So here upon a common road (not a royal one) Kilometer and I met face to face. From that friendly meeting of a member of the metric system a desire was kindled within me to know more about the family history of the meter, and since that time (1913) I have picked up some interesting facts covering the whole family, some of which it is my purpose to relate at this time.

Mother Earth is the mother of the meter. The French scientists who devised this unit of measurement at the close of the 18th century planned that it should not be dependent upon any particular measuring stick of human construction, but instead should bear a definite relation to the dimensions of our globe. The art of measuring the earth and determining its shape, known as *geodesy*, had already made much progress and no other people had contributed so much to it as the French. Therefore, it was not strange that the authors of the metric system should have decided to take as the basis of this system a meridian of the

earth's surface—a circumference passing through both poles. For greater exactness (as meridians are not all of precisely the same length) they selected a particular part of a particular meridian. They decided that a meter should be the ten-millionth part of the distance from the equator to the North Pole, measuring on the meridian passing through Paris.

In considering the genesis of the modern metric system, as a universal system founded on an invariable standard and symmetrically and conveniently developed, it is necessary to go back to Gabriel Mouton, Vicar of St. Paul's Church, Lyons, who first proposed in 1670 a comprehensive decimal system having as a basis the length of an arc of one minute of a great circle of the earth. One minute of arc would give the length of a *milliare*, which would be subdivided decimally into *centuria, decuria, virga, virgula, decima, centesima, millesima.* The virga and virgula would be the chief units of the system corresponding to the *toise* and the *foot* then in use. This geometric foot (*virgula geometrica*) was further defined by Mouton as corresponding to the length of a pendulum making 3,959.2 vibrations in a half hour at Lyons. This proposition contained essentially the germ of the modern metric system and Mouton's suggestion of the pendulum was soon repeated by Picard (1671), and by Huygens (1673).

During the eighteenth century several schemes were proposed by scientists for the improvement of the weights and measures, and although they were brought to the attention of the French Government they did not meet with such approval as to secure their adoption. These various schemes were discussed and discarded without any definite action, and just as in later times, the difficulties attending the introduction of a new system were anticipated and feared. In fact Necker, in a report made to Louis XVI in 1778, speaks of the proposed reform of weights and measures with considerable diffidence. He writes, "I have occupied myself in examining the means which might be employed to render the weights and measures uniform throughout the Kingdom, but I doubt yet whether the unity which would result would be proportionate to the difficulties of all kinds which this operation would entail on account of the changing of values which would necessarily be made in a multitude of contracts, of yearly payments, of feudal rights and other acts of all kinds. I

have not yet renounced the project, and I have seen with satisfaction that the Assembly of Haute-Guyenne have taken it into consideration. It is in effect a kind of amelioration which can be undertaken partially, and the example of a happy success in one province would essentially influence opinion."

Let us turn the pages of history a hundred years and discern this same note of diffidence in the following statement of J. E. Hilgard, Assistant, United States Coast Survey, and inspector, United States standard weights and measures, who in response to a resolution by the House of Representatives submitted a report dated March 21, 1878, on the obligatory use of the metric system for government business:

"Not only are lands purchased from the public domain described in a simple decimal system of acres measured by square chains and decimals, but all the most valuable real estate, such as lots and streets in cities, has been laid off in this country in even feet, generally even tens of feet, as 50, 60, 80, 100, 150, etc. What adequate motive is there to change these expressions into terms which are necessarily fractional and in which those foreign nations whose convenience it is proposed to meet have no conceivable interest? What useful purpose is subserved by designating a building lot 24 by 120 feet in the form of 7.315 by 36.576 meters? It is the foregoing and similar considerations which lead the undersigned to doubt whether the international units of measures will ever wholly take the place of all others in our domestic transactions."

In this connection, since we have already spanned a century, let us push on another half-century to our own times and again witness the spirit of conservatism in regard to really using the metric system. The experience of the United States army is of interest. It led to United States Army Order No. 26, dated May 3, 1920, issued after the War Department had given long and careful consideration to the proposal to use metric weights and measures in the military service of the country, whose fundamental system is English.

"Units of weight and measure for the Army.—As an incident to the World War, metric units of weight and measures were necessarily used in instruction and training. As a result more or less confusion now exists as to what units should be used normally."

"Hereafter the customary British units of weight and measure (including the degree Fahrenheit) will, as far as practicable be used throughout the Army as they were before the World War."

The very latest metric legislation reveals that Congress is hesitant about taking a definite issue in regard to the metric system.

The Britten-Gillett Metric Resolution (H. J., Res. 10) introduced into the House of Representatives December 5, 1927 by Representative Britten of Illinois reads as follows:

"Whereas the Constitution of the United States of America provides in section 8, paragraph 5, that Congress has the power to fix the standard of weights and measures; and whereas there now exists a lack of uniformity in the weights and measures of the United States; and whereas these are not standardized with the weights and measures used by the vast majority of the nations of the world;

"Therefore be it resolved by the Senate and House of Representatives of the United States of America in Congress assembled, that the Department of Commerce be authorized and directed to conduct, within one year, a thorough investigation and study to determine the advisability of adopting metric weights and measures for general use in the United States. This department, after making such investigation and study, is hereby authorized to initiate and carry out, to such extent as may be deemed advisable, plans to encourage the general and common use in the United States of such system of weights and measures."

To return to our story of the early development of the metric system. With the changes wrought by the French Revolution it was possible to gain at the hands of the public consideration for rational ideas in science as well as in government and religion. It was Talleyrand, a bold and able leader, then Bishop of Autun, who brought a plan for reform to the attention of the National Assembly in April, 1790. He not only appreciated the necessity for a uniform system of weights and measures for France, but also the desirability of a system that would be truly international rather than merely the weights and measures of Paris. Through the efforts of Talleyrand the National Assembly rendered a decree on May 8, 1790, which was sanctioned by Louis XVI on August 22 of the same year. It reads in part as follows:

"The National Assembly, desiring that all France shall forever enjoy all the advantages which will result from uniformity of weights and measures, and wishing that the relation of the old measures to the new should be clearly determined and easily understood, decreed that His Majesty shall be asked to give orders to the administrators of the different departments of the Kingdom, to the end that they procure and cause to be remitted to each of the municipalities comprised in each department and that they send to Paris to be remitted to the Secretary of the Academy of Sciences a perfectly exact model of the different weights and elementary measures which are in usage.

"It is decreed further that the King shall also beg His Majesty of

Britain to request the English Parliment to concur with the National Assembly in the determination of a natural unit of measures and weights; and in consequence, under the auspices of the two nations, the commissioners of the Academy of Sciences of Paris shall unite with an equal number of members chosen by the Royal Society of London, in a place which shall be respectively decided as most convenient, to determine at the latitude of 45°, or any other latitude which may be preferred, the length of the pendulum (seconds), and to deduce an invariable standard for all the measures and all the weights; and that after this operation is made with all the necessary solemnity, His Majesty will be asked to charge the Academy of Sciences to fix with precision for each royal municipality the relation of the old weights and measures to the new standard, and to compose afterward for the use of the municipalities the usual books and elementary treaties which will indicate with clearness all these propositions.

"It is decreed further that these elementary books shall be sent at the same time to all the municipalities to be distributed, at the same time there shall be sent to each of the municipalities a certain number of new weights and measures which they shall distribute gratuitously to those who would be caused great expense by this change; and finally, six months only after the distribution, the old measures shall be abolished and replaced by the new.

"Finally, the Academy shall indicate the scale of division which it believes most convenient for all weights, measures and coins."

A committee of the Academy, consisting of Borda, Lagrange, Laplace, Monge, and Condorcet, presented a report on March 19, 1791, to the effect that after an arc (of a meridian) had been measured the length of a quadrant could then be computed, and one ten-millionth of its length could be taken as the base or fundamental unit of length. The plan proposed was to measure an arc of meridian between Dunkirk, on the Northern coast of France, and Barcelona on the Mediterranean Sea, largely because these two places were each situated at sea-level in the same meridian, because they offered a suitable intervening distance of about 9° 30′, the greatest available in Europe for a meridian measurement, because the country so traversed had in part been surveyed previously by LaCaille and Cassini in 1739–40, and furthermore because such an arc extended on both sides of latitude 45°.

On May 29, 1793, in the report of the committee to the National Academy the name *Metre* (meter) was assigned to the ten-millionth part of the quadrant of the earth's meridian. The *provisional* meter was devised from a calculation of the operations made by LaCaille in 1740. A standard of the provisional meter in brass was duly constructed by Lenoir in Paris, and is

preserved in the Conservatoire des Arts et Metiers at Paris. This provisional meter varied from the meter finally determined by the commission in 1799 by only $\frac{1}{3}$ of a millimeter, being that much too long.

The King of France, on June 10, 1792, issued a proclamation, in which Delambre and Méchain, the two engineers entrusted to survey the arc between Dunkirk and Barcelona, were commended to the good offices of government officials and citizens generally, and various rights and privileges were secured to them. Both scientists straightway proceeded to their duties, but, owing to the turbulent conditions in the country, due to the Revolution they encountered from the beginning constant embarrassment and difficulties. In addition to being arrested and deprived of ordinary facilities to carry on their work, they met with little sympathy and cooperation on the part of officials and people, and experienced great difficulty in erecting and maintaining their signals, which were oftentimes believed to have been built for military purposes.

Méchain in Spain had a certain amount of assistance from the government of that country, but here, as in southern France, he was harassed and interfered with by political troubles. In fact, these two resolute engineers experienced almost incredible difficulties, being arrested by the various governing bodies that were at that time successively administering the affairs of France, deprived of liberty and freedom, prevented from working by accident and disease, and, in short, accomplishing most creditable results under remarkably adverse circumstances. Finally, in November, 1798, Méchain and Delanbre completed their survey and brought their records to Paris. Several committees were set to work checking results and compiling reports. On June 22, 1799, a platinum meter was adopted as the true meter, and was deposited in the Archives of the State, where it has come to be known as the Meter of the Archives. Thus we see it took seven long and arduous years to make the first standard meter stick. The units of mass and capacity were constructed along with the meter but space at present will not permit a detailed account of the evolution of these standards.

With the scientific determination of the standards there remained to effect the general adoption of the new weights and measures. This was a much more difficult task than at first

contemplated. After more than a third of a century of confusion and chatter among people of all estates with regard to the metric system the government was forced to act with determination and the following Act was passed, after much discussion, by the Chamber of Peers and the Chamber of Deputies, and was broadcast to the people July 4, 1837.

(In part) "After January 1, 1840, all weights and measures, other than the weights and measures established by the laws of 1795 and 1800, constituting the decimal metric system, shall be forbidden under the penalties provided by article 470 of the Penal Code. Those possessing weights and measures, other than the weights and measures above recognized, in their warehouses, shops, workshops, places of business, or in their markets, fairs, or emporiums, shall be punished in the same manner as those who use them, according to article 479 of the Penal Code. Beginning at the same date all denominations of weights and measures other than those authorized are forbidden in public acts, documents, and accouncements. They are likewise forbidden in acts under private seals, commercial accounts, and other private legal documents, etc."

It would be interesting to trace the development of the metric system for the next half century, including the establishment of the International Bureau of Weights and Measures at Sèvres near Paris. But all of the family affairs cannot be aired at the first telling so we shall leave France and return to our own United States and present a few real secrets about our standards of length and mass.

Washington, in his message to the First Congress, said: "Uniformity in the currency, weights, and measures of the United States is an object of great importance, and will, I am persuaded, be duly attended to." Acting upon this suggestion the House of Representatives referred the matter to Thomas Jefferson, who was then Secretary of State, for a report, and in July, 1790, he submitted two plans, one of which proposed to define and render uniform the existing system; the other "to reduce every branch of the principal affairs of life within the arithmetic of every man who can multiply and divide plain numbers."

Washington, in the opening address to the Second Congress, again urged the necessity for action but no legislation followed.

After the War of 1812 the subject was again taken up in compliance with an urgent appeal of President Madison. The Senate referred the matter for a report to the Secretary of State, John Quincy Adams, who, after much study, made a report at the conclusion of which he submitted his plan, which was:

(1) To fix the Standard with the partial uniformity of which it is susceptible, for the present excluding all innovation.

(2) To consult with foreign nations for the future and ultimate establishment of universal and permanent uniformity.

As before, no legislation resulted, and in consequence the weights and measures lawful in Great Britain during our colonial period remained in customary use in this country.

However much confusion might be tolerated in the weights and measures used for commercial transactions it was found necessary to legalize some particular weight for the use of the Mint, and this was done by Act of Congress, May 19, 1828; in the following language:

"For the purpose of securing a due conformity in weights of the coins of the United States to the provisions of this title, the brass troy pound weight procured by the Minister of the United States at London in the year 1827, for the use of the Mint and now in custody of the Mint at Philadelphia, shall be the Standard Troy Pound of the Mint of the United States, conformably to which the coinage thereof shall be regulated."

As the result of much public agitation, and on the first recommendation of a Committee of Congress, which submitted an elaborate report, and of which Hon. John A. Kasson was chairman, Congress passed the following Act on July 28, 1866:

"An Act to authorize the use of the Metric System of Weights and Measures."

"Be it enacted by the Senate and the House of Representatives of the United States in Congress assembled, That from and after the passage of this act it shall be lawful throughout the United States of America to employ the weights and measures of the metric system, and no contract or dealings, or pleadings in any court, shall be deemed invalid or liable to objection because the weights or measures expressed or referred to therein are weights or measures of the metric system."

Section 2 of the act gives the tables of conversion from metric to common units and this definitely states that 1 meter = 39.37 inches.

On May 20, 1875, a metric convention was signed at Paris by the representatives of seventeen Governments, among them that of the United States, for the purpose of establishing and maintaining, at common expense, a scientific and permanent International Bureau of Weights and Meaures near Paris (Sèvres). From this International Bureau the United States received her fundamental standards of length and mass in 1890.

In closing, let us follow the journey of these fundamental standards from the Old World to the New. Dr. B. A. Gould, delegate from the United States to the International Conference of Weights and Measures, held at Paris, September, 1889, officially representing our Government accepted the standards from the International Bureau. He had them packed and sealed and then transferred to the care of Mr. Whitelaw Reed, the United States Minister to Paris. From him Meter No. 27 and Kilogramme No. 20, and also Meter No. 12 of the alloy of 1874, were received by Professor George Davidson, Assistant, United States Coast and Geodetic Survey, by whom they were brought to Washington with great care and deposited in the Office of Weights and Measures. On January 2, 1890, Meter No. 27 and Kilogramme No. 20 were carried to the Cabinet room in the Executive Mansion, where the ceremony of breaking the seals upon the boxes was performed in the presence of the President of the United States, Benjamin Harrison, the Secretary of State, James G. Blaine, and the Secretary of the Treasury, William Windon, together with a distinguished company of scientific men. A formal certificate declaring the condition of these standards at the opening of the boxes was signed by the President and witnessed by the Secretary of State and the Secretary of the Treasury. A somewhat similar certificate was signed by the other gentlemen present. In consequence of this official act of the President of the United States, Meter No. 27 and Kilogramme No. 20, will be guarded as our National Prototype meter and kilogramme.

These National Standards are at present kept in a subterranean vault of the Bureau of Standards in Washington.

Strange as it may seem to those not accustomed to the science of exact measurement (metrology), Meter No. 27 is not a meter long, it falls short a bit. If you really want to know how long it is here is the equation:

Prototype No. $27 = 1\ \text{m} - 1.6\ \mu \mp 0.1\ \mu$ at 0° C., where $\mu = .000001$ m.

LOCOPHOBIA: ITS CAUSES AND CURE

BY GEORGE H. SELLECK

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Of the forty-eight questions on the College Board Mathematics *C* examinations during the last eight years, five have been locus questions. Thus the examiners have, intentionally or not, rated the study of locus as ten per cent of the study of geometry. At first thought this seems too high a value to set, but a careful consideration of the frequency of the use of loci in the construction of figures and the proof of problems perhaps justifies their valuation. These loci have not been especially hard, but they have obviously caused the candidates more worry than any other questions and have been treated more unsuccessfully.

The locus question of 1924 was: "*Two lines, AB and AC, each two inches long, form a right angle, BAC. Construct the locus of the points whose least distance from the figure is one inch.*" The construction to be performed consists of a combination of two of the simplest fundamental loci, the given lines being produced through *A* and two perpendiculars being drawn through *B* and *C* to show the limits of the parts of the locus. An amazing variety of diagrams was drawn by the candidates to represent this locus, many of them having no apparent bearing on the case. With the help of the other readers, Mr. Paul W. Waterman, of the Milwaukee Country Day School, made a collection of no less than eighty-five distinct figures which the writers of the papers thought correct. Figures which had been crossed out he ignored, or the number would have been considerably greater. Again in 1926 he collected eighty-one figures which had been drawn to illustrate the question: "*Find the locus of points from which the tangents to a given circle of radius 5 inches are 8 inches long. Give proof.*" Many of these diagrams resembled architectural drawings much more than anything else.

In 1925 the locus question was a starred proposition, and the variety of figures was smaller, but this was the first question, and many candidates did not recover their self-control suffi-

ciently to do much with the remaining questions. As a result the readers were obliged to make certain allowances to bring the number of passing papers up to about the usual record.

From the foregoing facts it is evident that there is something radically wrong with the study of loci in the secondary schools. Either the subject is too hard, or there is something wrong in the attitude of the student mind towards locus questions in general; and it looks as if a widespread condition of locophobia, or fear of the locus, exists, not only among students but also to some extent among teachers. This is surprising, because much graph work is now done in the algebra classes. This is usually enjoyed even in the class of beginners, and there is not an undue feeling of mystery present. The only method of procedure followed is first to plot some points of the graph, connect them by a straight line or smooth curve as the case demands, and second to show that the construction is right because all other points on the graph satisfy the given equation. The class has done good work, sometimes with much pleasure. The subject of locus has been well taught without the bother of definition as "*the path of a point moving in accordance with some given geometrical condition.*" The schoolboy is inclined to wonder why the mysterious point should ever move, and, if it should move, how, with its entire lack of size, could it ever generate an area or a volume for a locus. A locus is better defined as "*the place where all points satisfying a given geometrical condition, and no others, are located.*"

In the algebra class the first five fundamental loci could well be studied. The circle is regularly studied. First the curve is plotted. Second it is shown by the Pythagorean theorem that any point on the circumference is at the given distance from the center, which is generally at the origin. The perpendicular bisector, could be plotted, and the locus proved by the Pythagorean theorem. The locus of points equidistant from such lines as $x = 3$ and $x = -5$, or at a given distance from $y = 2$, could be fully treated. The bisector of a right angle could also be taken up, and the Pythagorean theorem used as proof. There can be no objection to the use of this proposition, because it is well known to all algebra classes. The proof of the theorem should be deferred until the proper place in the study of geometry or it might be proved informally.

An examination of seven of the more recent geometries shows an average content of 301 pages with the study of locus beginning on page 162. Then follows a five or six page chapter on loci with only an occasional locus in the rest of the book. In some books a complete locus proof is not anywhere assembled, but references are made to two theorems proved seventy-five or a hundred pages before. A two-way proof is either stated as necessary for the treatment of all locus questions, or it is implied. The fundamental loci are not sufficiently stressed, and there is nowhere a suggestion that only a one-way proof is necessary when a fundamental locus can be quoted as authority. Furthermore, the order of treatment of loci used in the graph work is usually reversed.

Chauvenet, Wentworth, Hobbs, and Wentworth and Smith begin the study of locus on pages 22, 45, 59, and 73 respectively, and from thirty to seventy or eighty locus questions appear scattered through the first three books. With the preliminary study of loci as graphs in algebra, it is easily possible to take up the locus very soon after the beginning of the formal study of geometry, and it can be continued through the whole course by introducing review exercises and by calculating lengths of loci dependent on the circle and the areas enclosed by them. In this way the present three per cent treatment of the later books can be expanded to meet the ten percent requirement of the College Board examiners, and a very efficient tool will be put into the hands of the student, the use of which he will enjoy.

In recapitulation the following suggestions are made:

(1) Put a good definition of locus in the algebra.

(2) Study graphs as loci, perhaps introducing some of the fundamental loci of geometry.

(3) Begin the study of loci very early in the geometry work and keep it up all through.

(4) Reverse the order of proof to conform with the algebraic treatment of graphs.

(5) State that, except in the fundamental loci, a one-way proof is generally sufficient, although it is sometimes necessary to define the limitations of the fundamental loci used.

(6) Talk about locus as often as possible. A locus a day keeps phobia away.

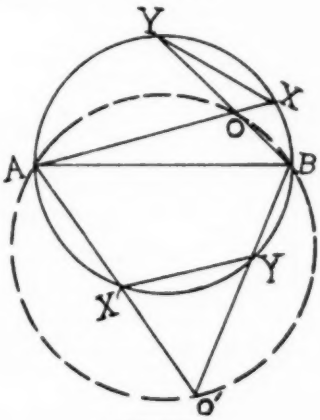
In the first of the following demonstrations a one-way proof

based on a fundamental locus is used. In the second, one suggestion, number 4 is followed with a consequent reduction to a minimum of the first part of the proof.

Problem. *AB is a diameter of a circle, fixed in position, and XY is a chord of a given length, but of variable position. Find the locus of the intersection of AX and BY.*

Solution. Let AB be the diameter of a $\odot$ in a fixed position, and XY be a chord of a given length of variable position.

To find the locus of the point of intersection of AX and BY.



As XY takes all possible positions, AX and BY will intersect at some point O, between AB and XY, and at another point O', outside the given circle.

Let n represent the number of degrees in the arc XY.

Statements	Reasons
1. $\angle AOB = \frac{(180 + n)^\circ}{2}$	1. An angle formed by two intersecting chords is measured by half the sum of the intercepted arcs.
2. $\angle AO'B = \frac{(180 - n)^\circ}{2}$	2. An angle formed by two intersecting secants is measured by one half of the difference of the intercepted arcs.
3. Therefore O and O' are on the arcs of the segments of a circle constructed on AB as a chord in which angles AOB and $AO'B$ can be inscribed.	3. The locus of the vertex of a triangle having a given base and an angle of a given size opposite, is the arc of the segment of a circle constructed on the base as a chord, in which the given angle can be inscribed.
4. Angles AOB and $AO'B$ are supplementary by 1 and 2 above, and therefore the whole locus is the circumference $AO'BO$.	4. A quadrilateral is inscriptible in a $\odot$ if its opposite $\measuredangle$ are supplementary.

Problem. *To find the locus of points from which tangents to*

a circle are equal to the distance to a given point outside the circle.

Given the circle with center A, and the external point B.

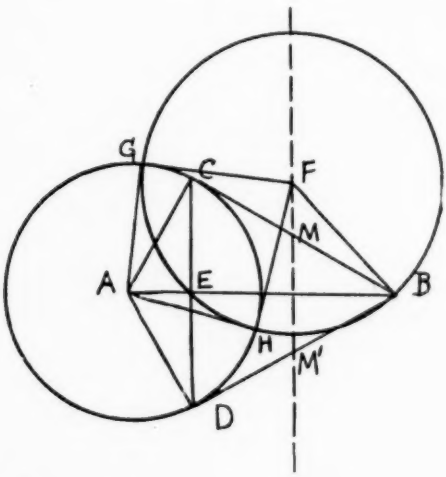
To find the locus of points from which tangents to the circle equal the distance to B.

Solution

First Part

Draw tangents BC and BD to the circle. Draw a line through M and M', the midpoints of the tangents. Also draw AC, AB, AD, and CD.

M and M', which satisfy the given conditions, are on the locus whatever its shape.



Second Part

1. $AC = AD$ and $BC = BD$.	1. Radii of a circle are equal, and tangents to a circle from a point are equal.
2. BA cuts CD at E, and is the $\perp$ bisector of it.	2. Two points equidistant from the ends of a line determine its perpendicular bisector.
3. MM' is $\parallel$ to CD.	3. The line joining the mid-points of two sides of a triangle is parallel to the third side.
4. MM' is $\perp$ to BE and bisects it.	4. A transversal of two parallel lines which is perpendicular to one of them is perpendicular to the other; and a line bisecting one side of a triangle, and parallel to another, bisects the third.

5. $\overline{AC}^2 = AB \cdot AE$.	5. Either leg of a right triangle is the mean proportional between its projection on the hypotenuse and the hypotenuse.
6. With any point, F, on MM', and radius FB, draw a $\odot$. It will pass through E, and cut the given $\odot$ at G.	6. MM' is the perpendicular bisector of BE.
7. $\overline{AG}^2 = AB \cdot AE$.	7. Substituting equals for equals in 5.
8. AB is a secant of $\odot$ $GE'B$.	8. By 6.
9. AG is tangent to the $\odot$ $GE'B$.	9. By 7.
10. AG is $\perp$ to the radius FG.	10. A radius to the point of tangency of a tangent is perpendicular to it.
11. FG is tangent to the given circle.	11. Converse of No. 10.
12. $FG = FB = FH$.	12. Radii of a $\odot$ are equal.
13. The line through M and M' is the locus of points from which tangents to the given circle equal their distance from B.	13. By the last paragraph of part one and 12.

By revolution about an axis, or by movement in a fixed manner, nine space loci may be developed from the first five fundamental loci.

I. *The locus of points equidistant from the ends of a line is the perpendicular bisector of it.*

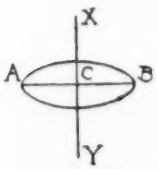


FIG. 1

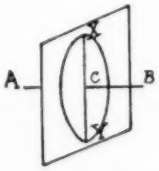


FIG. 2

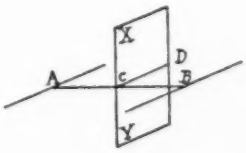


FIG. 3

By revolution about the perpendicular bisector, the locus of points equidistant from a given circumference is generated (Fig. 1); by revolution about the given line the locus of points in space equidistant from two given points is generated (Fig. 2).

By drawing CD perpendicular to AB and XY at C, the intersecting lines determine planes. Then by moving the lines AB and XY always perpendicular to CD, and XY in the plane XD, the locus of points in space equidistant from two parallel lines is generated (Fig. 3).

II. *The bisector of an angle is the locus of points equidistant from the sides of the angle.*

BE, drawn perpendicular to the plane of the $\angle ABC$ at *B* determines with *BD* a plane *DE*. By moving $\angle ABC$ so that *BD* is always perpendicular to *BE* in the plane, keeping the plane $ABC \perp$ to *BE*, the locus of points equidistant from the faces of a dihedral angle is generated.

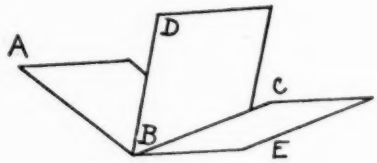


FIG. 4

III. *The locus of points equidistant from two lines of indefinite length is a line parallel to them and midway between them.*

IV. *The locus of points at a given distance from a line of indefinite length is a pair of lines parallel to it and at the given distance from it, one on each side.*

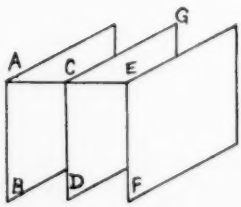


FIG. 5

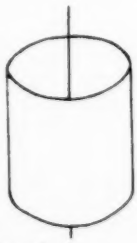


FIG. 6

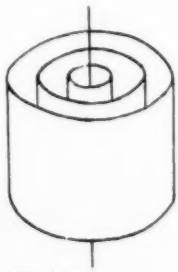


FIG. 7

AE, drawn perpendicular to *CD*, the middle one of the three parallels, is perpendicular to all of them. *CG* perpendicular to *AE* and *CD*, determines with *CD* a plane perpendicular to *AE*. If *AB*, *CD* and *EF* move with *CD* always in the plane *DG*, and with plane *AF* perpendicular to *CG*, two loci are generated: the locus of points in space equidistant from two parallel planes and the locus of points at a given distance from a given plane (Fig. 5).

If the three parallels are revolved about the middle one as an axis, a cylindric surface is generated, the locus of points in space at a given distance from a given line (Fig. 6).

If the three parallels are revolved about a line parallel to them in their plane as an axis, the locus of points at a given distance from a given cylindric surface is generated (Fig. 7).

V. *The locus of points in a plane at a given distance from a point is the circumference of a circle with the point as a center and the given distance as a radius.*

By revolving this locus on a diameter as an axis, a spherical surface is generated, which is the locus of points in space at a given distance from a given point. (Fig. 8.)

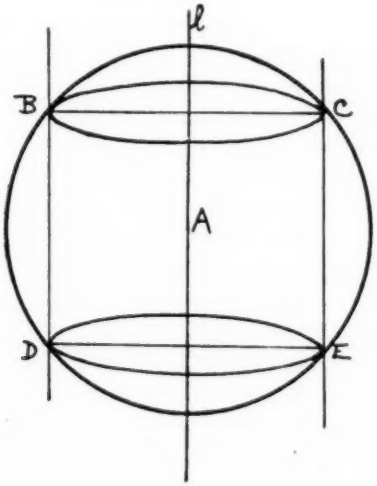


FIG. 8

Sometimes combinations of space loci can be easily developed from fundamental loci by revolution about an axis. The locus question on the College Board Mathematics *D* paper in 1923 was of this kind. It read as follows: "*Given a line l and a point A on the line. Describe and draw the locus of points which are always 10 inches from A and 8 inches from l.*"

A combination of fundamental loci III and V determines the points *B*, *C*, *D*, and *E* in a plane which are 10 inches from *A* and 8 inches from *l*. Revolving the whole figure on *l* as an axis generates two circumferences with centers on *l* and diameters *BC* and *DE* in planes perpendicular to *l* at points 6 inches from *A* as the locus required.

SELECTING TESTS FOR MEASURING ACHIEVEMENT IN HIGH SCHOOL MATHEMATICS

BY MINERVA DESING

University of Pennsylvania

In the early days of education, testing was almost a sort of contest between the teacher and the student. The teacher chose his questions with malice aforethought, and the student, in a mental perspiration, sought to acquit himself with as little dishonor as possible. The test offered defiance to the pupil to prove that he knew whereof he spoke. The attitude of both pupils and teachers has undergone considerable change since those days. The introduction of new-type examinations has gone far towards eliminating the fear of tests and the actual physical distress of testing periods from the experience of pupils. But we are here concerned with the attitude of the teacher. For her also testing has taken on a new meaning. The test is no longer an "instrument of torture," but a teaching device. We do not test to prove how little the pupil knows, but to determine what he knows and if possible why he does not know more. Education has been defined as the effecting of desirable changes in the pupil. To do this we must put knowledge, ideals, attitudes and the like where there were none before, or modify those already existing. It then becomes necessary for us to know what is in the minds we are to change. And this is to be discovered by testing alone. It is of no avail to teach the student what he knows. We must teach him what he does not know. And in order to keep track of the changes that have been brought about, we must resort to frequent testing.

For this purpose of mental stock-taking we must select good tests. What constitutes a good test? The first essential is that it test what it purports to test—that it possess validity. It should cover thoroughly what has been taught and avoid what has not been taught. In addition to including a wide range of facts, a good test should measure attitudes and similar objectives to some extent, if only to inform the teacher of her

success or lack of success in attaining the less apparent goals for which she is striving. I say to some extent, because we have thus far found it impossible to measure such things to any adequate degree. Even in the field of mathematics, where scientific methods are most easily applicable, we find ourselves falling far short of our goal in this respect. In the available standardized tests little attention has been paid to the pupil's ability to make and interpret a formula. We do our best to teach the child that a formula is an expression of relationship in algebraic shorthand, yet we do not try to discover whether or not he is able to apply this concept. We tell him that when the scientist has given us a formula we have the means to predict the result of a certain combination of circumstances without understanding the underlying technical considerations, and yet how do we know that $s = \frac{1}{2} gt^2$ is anything but another example to him? We introduce bits of the history of mathematics, we tell our students of unsolved problems, but we do not know whether or not they are oriented in the history of mathematics, whether or not they realize that mathematics and civilization march along side by side, that both are centuries old, and that the twentieth century does not mark the glorious completion of the quest for knowledge. It is not because these things are unimportant that they have found so little place in testing, but because of the extreme difficulty involved in adequate measurement.[1] The validity of a test for the particular situation in which it is to be used can be roughly determined by the teacher in reading it over. However, in order to understand the statistical data offered, some knowledge of methods of validation is necessary. A great many different methods are in use, but it will suffice to discuss a few of them briefly.

Text-book analysis, analysis of courses of study, and pooled judgment of experts may be regarded as essentially the same method. The test is validated according to frequency of occurrence of the items in one or more of these criteria. For instance, if ten leading text-books are analyzed, and types of questions tabulated according to frequency of occurrence, a test composed of the items of highest frequency will be quite representative of the kind of material taught. This is not the best

[1] See "What the Tests Do Not Test," by H. M. Walker, MATHEMATICS TEACHER, 18: 45–53 (Jan. 1925).

method of validation, but for subjects of recent introduction it is the only method. It has been severely criticized for encouraging a static tendency, a valid criticism, yet one not likely to carry much weight with 90 percent of teachers.

Social utility, an important factor in curriculum construction, is the basis of another less widely used method of validation. This method has the advantage of encouraging progress, as it suggests desirable changes in the course of study.

If we have access to test scores, ranks, or estimates of pupils in which we may place much confidence as a criterion of validity, we may take as a measure of validity the coefficient of correlation between the scores made on the test and scores, ranks, or estimates of the ability of the same pupils on the criterion. The size of this coefficient of correlation obtained will depend on the *reliability* of the test and also of the criterion. We may determine the correspondence between the test and a perfectly reliable criterion by making use of the correction formula

$$\bar{r}_{tc} = \frac{r_{tc}}{\sqrt{r_{cc}}},$$

in which $\bar{r}_{tc}$ is the correlation between the test scores and "true" criterion scores, r_{tc} is the observed or calculated correlation, and r_{cc} is the reliability coefficient of the criterion. For example an obtained coefficient of validity of .68 becomes .85 when corrected as above for a reliability of .64 in the criterion.

One of the most important features of a test is its reliability, that is, consistency in score. If there is more than a single form of the test, the reliability may be determined by correlating the scores obtained by the same pupils on equivalent forms. If only one form is available, the reliability is determined by correlating the scores when only even questions are counted with those when only odd are considered. If the test is correlated by this method, the coefficient thus obtained for the half-test will be less than the coefficient for the whole test as, other things being equal, the reliability of a short test is less than that of a long test. It is therefore necessary to make a correction which may be computed by use of the Spearman-Brown prophecy formula, which gives

$$r_{11} = \frac{2r_{\frac{1}{2}\frac{1}{11}}}{1 + r_{\frac{1}{2}\frac{1}{11}}},$$

in which $r_{1/2\ 1/11}$ is the correlation between scores on odd and even questions, and r_{11} is the true reliability of the entire test.

It now remains to examine the significance of the reliability coefficient. In itself it is not a sufficient index of reliability. The first thing to be considered is the range of ability over which the computation was made. The greater the range, the greater the coefficient that will be obtained, so that the range should be given also. Computation based on the scores of grade IX may give $r = .60$ while computation over grades IX to XII may give $r = .90$. The values are related according to the formula

$$\frac{\sigma_1}{\sigma_2} = \frac{\sqrt{1 - r_2}}{\sqrt{1 - r_1}},$$

where the subscript 1 indicates one group and the subscript 2 the other. Another cause of the inadequacy of r as an absolute measure of reliability is that it does not, by itself, indicate the probable error of the individual score. This may be computed, for raw scores, from the formula

$$\text{PE}_{\text{meas}} = .6745 \left[\frac{\sigma_1 + \sigma_2}{2} \right] \sqrt{1 - r_{12}}$$

if both sigmas are known, or from

$$\text{PE}_{\text{meas}} = .6745\sigma\sqrt{1 - r_{12}}$$

if only one sigma is available. We are now able to say that the chance that the pupil's true score lies between the limits of the score $\pm$ P.E. are 50 out of 100. The following table of acceptable coefficients of reliability is given by Ruch and Stoddard.[2]

Reliability Coefficient	Interpretation or Significance
0.95 to 0.99	Very high; rarely found among present tests
0.90 to 0.94	High; equaled by a few of the best tests
0.80 to 0.89	Fairly high; fairly adequate for individual measurement
0.70 to 0.79	Rather low; adequate for group measurement but not very satisfactory for individual measurement

[2] Ruch, G. M. and Stoddard, G. D., *Tests and Measurements in High School Instruction*, page 56.

Below 0.70.....Low; entirely inadequate for individual measurement although useful for group averages and school surveys.

McCall states that a reliability coefficient of .55 is sufficient for a class of ordinary size, while a coefficient of .90 is required for important individual diagnosis. These figures should be regarded as based on a range of ability as represented by one school grade.

Directions and ease of administration are other points to be considered in selecting a test. Specific directions for administration should accompany the test. Directions to the class should be clear and brief, for lengthy directions are confusing. They should, however, be sufficiently comprehensive to establish standard testing conditions. Samples and fore-exercises should be included where necessary.

Another important characteristic of a good test is its relative objectivity. Tests are used extensively for the purpose of marking. How efficiently do we mark when what is accepted from one pupil is not accepted from another? (Here I assume students of comparable ability, for standards should vary with ability.) How efficiently do we mark when a pupil's ranking depends on how the teacher feels when marking the papers, or on whether or not the student comes to school with a well scrubbed face? There should be one answer, and only one, to each question, so that whenever and by whomever the paper is marked, the score will be the same.

The scoring key should be convenient to use. Adequate directions should be included. The score card of a certain test, one part of which tests the vocabularly of arithmetic, provides a good example of what a score card should not be. For some questions it accepts synonyms, for others it does not; in some cases reversed order is acceptable, while in others, where it should obviously be permitted, no directions to that effect are given; and although it tests vocabularly, no mention is made of the acceptability of misspelled words. This immediately lowers the objectivity because some teachers will use their judgment, and others will do nothing without specific directions.

One of the best features of standardized tests is the presentation of norms. On the basis of the maturity of the pupil, there are two kinds of norms, grade and age norms. These are mean,

median, or percentile scores of a large number of cases of a given grade or age. Although age norms are more generally useful than grade norms, they have little significance for tests in high school mathematics. All norms should be accompanied by some measure of spread, such as the quartiles. Percentile norms are now coming into favor. They permit a closer comparison of the entire class to norm than do norms of the mean or median types. The number of cases upon which a test is standardized should always be given. Median norms should be based on at least several hundred cases, while a larger number is required for percentile norms. But more important than the number of cases is the factor of selection. The cases upon which the norms are based must constitute an absolutely random selection. If the standardization is upon results from a single locality, or upon voluntary returns from teachers using the test, the selection is not random. Norms should be computed on results from a number of school systems throughout the country, but it is helpful to include "locality norms" also. Another point to be considered is the time of year for which the norms are derived. If the test is to be administered at any other time, interpolation will be necessary.

Some tests are accompanied by directions for interpretation, and have remedial exercises which are easily procured. These save the teacher much time, and are likely to be more effective in overcoming difficulties than exercises devised by the teacher herself.

Other things being equal, a test with a number of "equivalent" or "similar" forms is to be preferred to a test having but one, as it is essential to the measurement of progress that equivalent tests be given at later dates, and the practice effect makes it inadvisable to give the same form twice. The forms should be equivalent, that is they should cover the same ground, they should be of equal difficulty, and standard deviations and means of scores made by the same group of pupils on two or more tests must be approximately equal, if they are to be used as similar forms.

The final consideration is economy. Economy of time, labor, and money is an important criterion in test selection. If a test requires more than forty minutes to administer it is difficult to give in a high school period, and is too fatiguing to be a fair

test. On the other hand, the longer the test, other things being equal, the more reliable. The work of a term should not be measured by a test of ten or fifteen minutes. The time required for scoring is an important matter to the teacher. The system of partial credits used by some authors tends to complicate the scoring process, but most tests are rapidly scored because of the devices used.

The following references will be found helpful in the selection of tests:

KELLEY, T. L.: Interpretation of Educational Measurements. World Book Company, Yonkers-on-Hudson, N. Y., 1927. Pages 326–328.

Excellent detailed classifications and ratings of available tests.

RUCH, G. M., AND STODDARD, G. D.: Tests and measurements in High School Instruction. World Book Company, Yonkers-on-Hudson, N. Y., 1927. Chapter V.

Good description and discussion of tests.

SYMONDS, P. M.: Measurement in Education. Macmillan Co., N. Y., 1927. Chapter VI.

Discussion and critical analysis.

THE FOURTH YEARBOOK

— OF —

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A FEW DETERMINANTS IN BUILDING THE COURSE OF STUDY IN MATHEMATICS *

BY GERTRUDE JONES

High School, Lincoln, Neb.

A casual survey of educational literature reveals the tendency of movements in education to come in waves. The project method, educational measurements, student government, and supervised study are a few of these waves that have been the chief concern of educators until a new wave has rolled in. At present, educators everywhere are riding the wave of curriculum reconstruction, the inevitable result of several factors, among which are the problems presented by the junior high school, the great increase in the senior high school enrolment with its consequent wide variation in ability and interests, and the rapidly changing conditions of modern life. This problem of the curriculum is one of vital concern. Professor Thomas Briggs calls the curriculum problem fundamental in education since its content determines the organization of the school, the types of buildings, the methods used, and the value of the school to the community.

One principle of paramount significance forms the basis of all curriculum building. No longer is knowledge or subject matter for its own sake the chief concern in the process we call education. Subject matter is to be used only as a means of developing and strengthening those desirable appreciations, attitudes, abilities, habits, powers, and skills which are considered essential to worthwhile living in the modern social scheme. Curriculum builders might well take for their slogan Emerson's saying "There is no knowledge that is not power."

To attempt to discuss fully within the limits of a brief paper the building of the course of study in mathematics would be impossible as well as absurd before a group of teachers who are familiar with the report of the *National Committee on Mathe-*

* A paper read before the Mathematics Section of the Nebraska State Teachers Association at a meeting held November 1, 1928.

matical Requirements and with the *Second Year Book of the National Council Of Teachers of Mathematics.* Therefore, let us consider a few points bearing on this subject which have suggested themselves to one who has been trying to teach mathematics in the senior high school.

National committees can point out general tendencies and they can make helpful suggestions, but they cannot map out a course of study that will meet the needs of every community equally well. This means that a local committee should first make a broad survey of all the community factors, and then lay out the general route from the elementary school through the college after it has determined the major objective of all mathematical education and the specific objectives for the mathematics to be offered in each of the several schools. Then the units of work may be selected, the test of each unit being its possible contribution toward the realization of the specific and general objectives which have been selected.

On such a committee should be represented the expert in curriculum building, the administrative staff, and experienced teachers of mathematics in the elementary school, the junior high school, the senior high school, and the college. The expert can contribute not only the latest thought in educational psychology and philosophy, but also the results of the work that is being done in the many experimental schools throughout the country. The help of the administration will prove valuable to the committee in many ways. No one is better fitted to suggest the actual units of work and their methods of presentation than the class room teacher of experience. In the mapping out of the general route we need the viewpoint, the cooperative effort, and the constructive thought of experienced and interested representatives of the various schools concerned. Teachers of mathematics should be familiar with the work that is being done in schools other than their own if they are to do their best teaching. The teacher in the senior high school knows what she expects of the pupils when they come to her, but is she familiar with the courses in mathematics which the pupils have studied? Teachers in the elementary school and in the junior high school know what they are trying to teach to their pupils, but do they clearly understand the problems which their pupils must meet in future years and for which they must therefore be prepared?

The actual building of the course of study in mathematics cannot be begun until a general objective has been set up for all mathematical study. It would be difficult to formulate a more comprehensive objective than the one stated in the report of the National Committee. I quote. "The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and a control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual." After all, isn't this statement a definition of what we might call "quantitative thinking"? Then the general objective of every teacher of mathematics from the elementary school through the college is the development of the power of quantitative thinking, a power which is needed increasingly in these days.

The teaching of mathematics in each of the schools must have its specific objectives, and these must be determined before the actual course of study is built. The primary objective in the elementary school is the development of the power to perform accurately the four fundamental operations with integers and with common and decimal fractions. Arithmetic drill should not cease with the elementary school, but should be continued every day as long as the pupil studies mathematics or these skills are likely to be lost to a certain degree. We do not expect a pianist to retain his technical skill without daily practice. The same principle applies to arithmetic skills. Since the building of a course of study implies method as well as content, it is the duty of every teacher of mathematics to insist upon absolute accuracy of computation and of expression. Pupils should be taught to estimate results first, then compute, and then check results, expecting no credit for method if results are inaccurate. Some will say that this is rather harsh treatment, but, on the other hand, a pupil has a motive for careful computation if he knows that only accurate results will be accepted. Allowing some credit for correct method is an excellent way to establish harmonious relations in the classroom, but after all, it is the formation of correct habits that should have the first consideration.

As to the objective for mathematics in the junior high school I quote again from the report of the National Committee. "In the junior high school, comprising grades seven, eight, and nine, the course for these three years should be planned as a unit with the purpose of giving each pupil the most valuable mathematical training he is capable of receiving in those years with little reference to courses he may or may not take in succeeding years." We note the tendency of junior high schools over the country to offer courses in general or unified mathematics—a sort of survey or pre-view course of the whole field of mathmatics. Such courses should prove valuable if it is remembered that too much breadth and too little depth is very likely in three years' time to breed shallowness and superficiality in the pupils. The teacher of mathematics in the senior high school has the right to expect that the junior high school pupil has not only surveyed, but that he has become master of what he has surveyed.

Many teachers are coming to believe that the junior high school should not attempt to give courses in formal algebra or in demonstrative geometry, but that these courses should be left to the senior high school where they properly belong. Some one has said, "In the junior high school we give the mind its *chance*, in the senior high school its *choice*." It seems, therefore, that the purpose of mathematics in the junior high school is two-fold. First, the pupils are to be given a chance to find out what mathematics is really like and to show whether they possess or lack mathematical ability. Upon these discoveries should rest the decision as to whether they should continue with the study of mathematics in the senior high school. The second objective is to build a foundation for future mathematical work whether it be in school or in the solution of problems outside the school. The majority of the courses in mathematics offered in the junior high school include arithmetic, intuitive geometry, the formula, graphic representation, directed numbers, the equation, simple cases of factoring, and numerical trigonometry. If the pupils not only survey these topics, but really learn them, they will possess a foundation upon which the senior high school teacher can build and which will make possible more intensive and more varied work in the senior high school. The junior high school is still in the experimental stage. Much more experimenting is necessary before we can state just exactly the

type of course in mathematics that best meets the needs of the junior high school.

The overwhelming increase in senior high school enrolment demands that courses be offered in mathematics for two types of pupils,—the college preparatory and the non-college preparatory groups. It is difficult to say who is suffering the more, the teachers or the pupils, from the present arrangement of placing both types of pupils in the same courses. In 1893 the Committee of Ten declared that "every subject which is taught at all in a secondary school should be taught in the same way and to the same extent to every pupil so long as he pursued it, no matter what the probable destination of the pupil or at what point his education is to cease." We have come a long way in our thinking since 1893, but our practice in mathematics still leaves much to be desired. Let us consider first the college preparatory group.

Many teachers believe that the study of formal algebra should be postponed until the tenth grade, which would still leave six semesters for the study of mathematics in the senior high school. With the foundation built in the junior high school plus an added year of maturity, we might expect to develop algebra students rather than the manipulators that algebra seems to produce. One great source of trouble with our algebra is the fact that too many pupils can "juggle" terms and get the correct result in a given situation, but they do not grasp the underlying principle. They perform operations mechanically, but they do not work thoughtfully. The pupil who can handle the equation $2x = 6$, but who fails to see that the equation $15x = 7$ involves the same underlying principle, does not really know how to solve an equation of the type $ax = b$. Algebra as well as geometry should be a course in thinking and in the expression of that thought in words.

Only two types of pupils should be permitted to enter classes in college preparatory mathematics in the senior high school—first, those who plan to become engineers or teachers of mathematics or those who are preparing for work in the sciences that require mathematics as a foundation; and second, those who actually have the ability to master mathematics and who like it for its own sake. The core of these college preparatory courses should consist of those things which the engineer or the student

of college mathematics should have constantly at his fingers' ends. These elements can be determined upon by reference to the syllabus issued by the *Society for the Promotion of Engineering Education* and by conference with college professors of mathematics. With a full knowledge of the points to be emphasized and with a more or less select group with which to work, the teacher of these courses should make it her business to see that each pupil masters these requirements to the best of his ability. It should then be the teacher's privilege to recommend that the pupils who have succeeded in mastering these elements creditably continue with mathematics in college, and likewise to advise those pupils who lack ability or who have not done creditable work not to continue with the subject. Thus, the professor of engineering and the professor of college mathematics would know definitely in what respects the pupils who come to him are prepared. When it becomes necessary to use mathematical methods more advanced than their preparation warrants the teacher will surely realize that his students are entitled to a careful development and explanation of these new points.

An examination of mathematical courses in the senior high schools reveals the tendency to offer trigonometry, analytical geometry, and even the calculus. However, there are those who believe that the senior high school should not attempt to teach these subjects; that if these more advanced subjects are offered, they should not be of college grade. Many teachers feel that it is much more consistent for pupils to master the elementary work than it is to "tinker" with courses of a more advanced school. It might be wise to abolish the classes in trigonometry in the senior high school and to substitute for them classes in high school algebra. This would reduce the number of pupils in the other algebra classes, and make possible more individual instruction with more efficient pupils in algebra, and consequently better results. In the place of the trigonometry, there might be substituted a course in review mathematics—a course which would secure individual initiative, a thing which would be possible with more mature students who have a background of algebra and geometry. A review of algebra and geometry with the function idea stressed throughout the course would enable the pupils to make the connections between the various topics

they have studied and thus obtain a well-rounded idea of their essential unity. If a pre-view course in mathematics is advisable in the junior high school, would not a review course in the senior high school be valuable?

And now, a suggestion or two as to courses in mathematics that might be offered to the non-college preparatory group. Is it wise to permit senior high school pupils to have no mathematics whatever for a period of three years? Many teachers of high school mathematics can remember the result of the fad not to teach any arithmetic in the eighth grade. No one denies the lack of wisdom of requiring every high school pupil to study formal algebra and demonstrative geometry. But would it not be wise to require at least one course in mathematics of every senior high-school pupil, preferably in the eleventh or the twelfth grade—a course in common sense mathematics or in the mathematics of common use, not a snap course, but a truly helpful course, its content to be drawn from real life problems. Such a course should be built on the needs of the community. It might include a review of the fundamental processes with integers and with common and decimal fractions, work with equations and formulas, graphic representation, percentage, interest, trade, measurements, investments, denominate numbers, and so on. Pupils in the solid geometry classes who reveal their lack of knowledge as to the number of pecks in a bushel or the number of feet in a mile, to say nothing of the pitiable inefficiency of the average high school pupil in arithmetic, point to the need of such a course.

As an elective for the non-college preparatory group, may I suggest a course in the appreciation of mathematics? We have courses in art appreciation, music appreciation, and the appreciation of literature, why not a course in the appreciation of mathematics? One can think of many fascinating topics that might be included in such a course. The history of our number system is something that every educated person should know. A study of the history of measurement might be included in such a course. How many pupils after six semesters of mathematics in the senior high school can discuss intelligently the metric system of measurement if they have not come in contact with it in some other science?

We might compare the broadening of the course of study in

mathematics to meet the needs of all the pupils to the building of a highway—a highway that starts with the elementary school, crosses the bridge which we call the junior high school, continues on through the senior high school and the college toward the ultimate goal of quantitative thinking. This highway is to replace the old traditional road which has proved to be too narrow and too full of ruts to accommodate modern traffic. This old road has been only a detour for many who have traveled its weary length, not knowing whither they were bound. The new highway must be broad. Its foundation must sink deeply into sound principles of psychology and philosophy. It will not last forever, but, like every well cared for public road, it must constantly be kept in repair. Old material will have to be dug out and new material inserted in its place. This highway must be designed to accommodate not only the "straight eights" and the "silent sixes" which take the bumps easily and reach their destination with little difficulty, but also the "two-ton trucks" that lumber along with much noise but little speed. There will be those who travel on foot or of poor understanding who will often have to sit on the curb and rest. There will be frequent service stations along the way furnishing up-to-date guide books and expert service men (except that in mathematics it is mostly women). Towing service will also be furnished when it is absolutely necessary, but the general plan of travel along this highway is for each traveler to know whither he is bound, why he is there, and to be able increasingly to diagnose the cause of trouble himself without having to send out an "S. O. S" for service. We hope that each one who travels this highway will develop certain desirable attitudes toward all traveling, that he will attain certain appreciations of what he has seen along the way, that he will, from this experience, develop certain powers and habits that will make future traveling, of whatever sort, easier for him. Above all, we hope that he will at least catch sight of the goal at the end of the road—quantitative thinking.

VARIABILITY AND FUNCTIONALITY IN HIGH SCHOOL MATHEMATICS

By LAURA BLANK

Hughes High School, Cincinnati, Ohio

The subject of variation as a topic or possibly phase of mathematics is a very important one in the field of the secondary school. In this day, when so much is made of correlations within the general field of mathematics and with other allied fields of science, the study of variation brings out numerous interesting contacts and inter-relations for the pupil, thus impressing him with the great breadth of application of mathematics. Moreover, gradual development of the idea and use of variation establishes certain modes of thought applicable to matters not wholly mathematical and perhaps only relatively quantitative. The concept of variability or functionality is perhaps sufficiently far-reaching in scope and application, if properly studied and emphasized to serve more than any other idea or agency to tend to correlate or unify mathematics.

The notion of variability is a very fundamental matter of life. A quantity changes under a certain condition or during a certain period or discussion. With its changes, as a direct consequence of these changes, another or several other quantities change. If the former magnitude is such that its values are observed or arbitrarily assigned, it is known as the "independent variable." The latter magnitude whose values correspond with, or grow out of the values of the independent variable is called the "dependent variable." However either magnitude is a function of the other. The change in the first magnitude may be continuous, in which case the corresponding change in the other related magnitude is probably continuous. At once one's scientific curiosity is challenged to discover precisely how the one variable will change with the other. Just how should the price of wheat vary with the size and quality of the crop of a given year? How should blood pressure, in a healthy person, vary with age? How does the number of articles of a certain sort sold vary with the

decrease in price due to greater production? Is the continuous decomposition of radium in accordance with a mathematical law? The idea of dependence of one variable upon another, whether referred to, or not, in class discussion as "functionality"—a word difficult of comprehension to young folk—is an important notion to understand and apply in practical situations of every-day life.

The use of the functional idea in connection with numerical computation work, which appears on the surface wholly numerical, as in the use of tables of squares or square roots or tables of simple interest is desirable as an introduction to the subject of variation. The pupil may not realize or may not need to realize that he is dealing with values of a dependent variable corresponding with, or constructed from x^2 in the equation $y = x^2$, with $\sqrt{x}$ in the equation $y = \sqrt{x}$ and with t in the equation $i = p \cdot r \cdot t$ where r and p are constant. Yet thorough familiarity with such tables due to use in many and varied abstract and concrete problems is much to be desired and all but necessary to their proper use.

During a first discussion of variation as such, prior to a lengthy matter of theorizing, concrete examples of relationship of great variety and number should be considered, such as the relationship of distance to rate and time, of interest to principal and rate, of area to length and width, of circumference to radius or diameter, etc. If the diameter is increased, what happens to the circumference? If the circumference is decreased what happens to the diameter? The diameter and circumference are variables each dependent upon the other. At this point in the development the constant of variation is ignored. A decrease is observed as a necessary consequence of a decrease and an increase a necessary consequence of an increase. The quantitative comparison or relationship is a later development. We note that the magnitudes themselves are not compared but their measures in certain comparable sorts of unit.

Then as a consequence of considering numerous particular examples of functionality, cognizance of variation as a general and abstract relation develops. Curiosity at once arises as to modes of change, or sorts of relation. There is the most fundamental type, direct variation, usually called merely "variation," that in which one quantity increases as another increases or decreases as another decreases. The quotient of the two variables,

or their ratio is a constant. If one quantity is doubled the dependent quantity is doubled. If one is tripled, the other is tripled. If one is divided by two, the other is divided by two, also. Again we resort to particular formulas to illustrate. This sort of discussion and analysis is a far more advanced and meaningful procedure psychologically than a mere automatic and elementary substitution of numbers for letters in a formula, though the formula might have been recognized previously as having meaning. Considering such a formula as the uniform distance formula, $d = rt$, regarding t as fixed, or, in other words, for a given time, if the rate is doubled, what happens to d? Or if d is trebled what occurs to r? Or in a later discussion, let the rate remain unchanged, let t vary and note the corresponding change in d, talking always in terms of rates of pedestrians, bicycles, automobiles, trains and airplanes, so as to make an abstract problem concrete for immature minds. The value to be obtained from this or similar formulas lies largely in the observation of the behavior of the distance when one or the other of the quantities varies, whereas a mere mechanical substitution of values so as to secure an answer and end the matter entails little real reasoning, perhaps only imitation.

Considering the formula, $C = 2\pi r$, at this point, perhaps one should deliberately digress. The radius and circumference are variables each dependent upon the other. But what is the significance of π? What sort of quantity or magnitude is π? Is it a variable? Because of its nature, π is a mathematical entity puzzling to young folk in their early secondary school work. In a discussion of formulas as part of the large general subject, variability, one should find an opportunity to clear up the obscure and erroneous ideas relative to π so as to put it in its proper classification. Moreover such a time is opportune since the study of variation presupposes an understanding and familiarity with the subjects of ratio and proportion.

After the study and use of direct variation should come the study of joint variation, though the use of the term "joint variation," except by classes pursuing contemporaneously courses in general science, physics, or chemistry, is not necessary. Any variation in which one variable decreases as the product of two or more others decrease, or in which one variable increases as each of two or more others increases is called a joint variation. Formulas such as $d = rt$ and $I = prt$ where all of the letters

are variable quantities are probably the most familiar examples of joint variation. The consideration of one of the variables of the product as increasing as the other decreases suggests situations for pupils, perhaps never considered by them in a quantitative way in connection with the formula or equation, though perhaps vaguely contemplated in such connections as automobile travel or savings accounts.

A third type of variation, by no means abstruse, is inverse variation, though it need not in an elementary course be known as such. When two quantities are so related that their product is a constant we say that one of them varies inversely as the other, or one of them is inversely proportional to the other. Or, considered in another way, any variation in which one variable decreases as another increases, or vice versa, is called an inverse variation. The example of the lever of which the formula is $w \cdot l = k$ or in the terms of a proportion: $w_1/w_2 = l_2/l_1$, is understood, though not precisely quantitatively understood, by every small child playing on the see saw of the playground. In fact the law is usually discovered experimentally by him, the exact mathematical significance being found by him probably only in the case of $l_1 = l_2$ as in the example of balance, the lever arms being of equal length.

There are, of course, other types of variation of more complex nature which might be considered. For example, the time required to dig a ditch varies directly as the length of the ditch in rods and inversely as the number of men working on the job. That is $t = kr/n$.

The most vivid and convincing instances of the idea of relationship or dependence of one quantity upon another occur in the study of graphs. The development of a table of values from a formula or equation, in which one of each pair is chosen arbitrarily, the other or dependent value following of necessity from the first selection, analyzes and reiterates again and again that correspondence which exists between related magnitudes briefly referred to by scientists as functionality. Again, in the actual process of plotting such pairs of values are emphasized the ideas of dependence and continuity of relationship. The discovery that direct variation produces straight lines and inverse or joint variation, curved lines graphically is significant and instructive. Conversely, the graphic representation of data of experimental origin if the successive values can be shown to be

continuous through a scale of values and represented by points in a straight line should be such as to be expressed mathematically by a first-degree equation. Moreover if the graphic representation of a set of related pairs of values obtained experimentally is a parabola or other conic section, and one is convinced of the continuity of the curve, the data should be such as to be represented in mathematical relationship by a second-degree equation. The example is one of inverse or joint variation.

Moreover the type of variation, if any of the known sorts, can be arrived at experimentally by finding the ratio of any pair of related magnitudes and comparing that ratio with the ratio of any corresponding pair. If these ratios are equal we have an example of direct variation. If these ratios are unequal, by finding the product of any pair of related magnitudes and comparing it with the product of any other corresponding pair, and finding these products equal we decide that we have probably an example of inverse variation. Moreover there is here always the question of continuity of pairs of values. However it should be shown that the problem of arriving inductively at a scientific law from a set of experimental data is often extremely abstruse and involved, often a problem impossible of solution. A population curve obviously represents some sort of relationship, but what it is has never been evolved.

The subject of variation has many contacts and inter-relations. It may be regarded as an aspect of ratio and proportion. We may convert a problem in variation into a problem in proportion. For if, for example, x varies directly as y, $x = ky$. But having the ratio of x to y in any particular instance and the ratio of x to y in another particular instance, we should have such a proportion as $x_1/y_1 = x_2/y_2$, for each of these ratios equals k, the constant of variation. The same is true of other sorts of variation, as for instance in the case of inverse variation in Boyle's Law: $v_1/p_2 = v_2/p_1$. As soon as pupils are reminded of all of the mathematical possibilities as applied to a proportion, of its being a particular equation, and hence subject to the operations of all of the algebraic axioms, applicable to equals, variation becomes an old familiar subject. The use of the constant of variation, k, in converting a statement of variability into an equation, and hence making of it a mathematical entity flexible under the algebraic axioms is for many of the older pupils a

clever device of the mathematician. The recognition, for instance, of the value of k as becoming π in the case of the formula for the circle and circumference familiarizes the pupil with the constant of variation, k, and its use in variability.

In one of its most elementary forms the subject of variation or functionality is merely a matter of numerical substitution. It may not involve even the use of an equation, but be only a series of substitutions with a comparison of the results obtained, a study of the behavior of the expressions secured by such substitutions.

Considering variability in geometric connections, if, for example, the area of a rectangle is regarded as a constant, the length varies inversely as the width. Or, regarding the length as fixed, the area varies as the width. Again, regarding the three angles of a triangle as constant, the area is a variable depending upon the length of any one side or of any one altitude. Considering an angle relative to its position with respect to a circle, the measure and hence the size of the angle varies as the vertex moves from the center of the circle to a point inside the circumference but not at the center, the angle then being formed by two intersecting chords, as the vertex moves to a position on the circumference, and finally as it moves to a position outside the circumference. Countless other examples of variability as evident in geometry could be cited.

Variability as applied to, or evident in, geometry and trigonometry is convincing because of its vivid and graphic nature. The variability of the signs of the trigonometric functions of an angle as one considers the angle increasing so that the terminal line moves from quadrant to quadrant, and the variability of the absolute value of these functions becomes most emphatic if reenforced by a drawing during the period of practice study so as to call up a mental picture of the functions with reference to a unit circle.

Let us consider with our classes of older pupils the mathematical method of scientific investigation or research in contradistinction to that of the student laboratory, which is usually deductive. A scientist observes certain phenomena. He sets up for himself problematic theory as to a variety of variation exhibited in his data. He writes a variable which his experiments suggest. Then he converts the variable by means of the device of a possible constant of variation into a formula or equation. Let us suppose that he sets up the hypothetical law: $d = k \cdot t$, d

being the distance of a body falling from rest and t the time consumed in its falling. He sets up apparatus to test his theory. He endeavors to eliminate extraneous interference so as to test his theory alone. Being convinced that there is no foreign interference in his experiment he reads his results for tabulation. He repeats the experiment many times, each time recording resulting data. Then he applies the results of his continued experimentation to the proposed law formulated for examination. Suppose the body falls 16.08 ft. in 1 sec., 64.32 ft. in 2 sec., and 144.72 ft. in 3 sec. He finds, upon substituting the first pair of related magnitudes, that $k = 16.08$. Upon substituting the second pair k is found to equal 32.16. Upon substituting the third, $k = 48.24$. But k was a constant, according to his theory. Hence the law proposed is invalid. Possibly d varies inversely as t. Tests by substitution utterly break down such an hypothesis. Possibly d varies as the square of t. Tests show this last law proposed to be valid. Needless to say, the scientist does not generalize or publish his law at once. He makes numerous experiments under different conditions before deducing his law conclusively.

The study of variation has many correlations and cross contacts with other topics in mathematics and other sciences. It is a study of simple substitutions, of formulas and hence of equations, of speculation and experimentation with proposed formulas, of graphic interpretations of formulas or equations, of scientific tables, and of graphs as yet not explained by formulas.

The subject of functionality or variation is not a subject to be treated in a brief chapter in the last high school course in mathematics, or in the third year, but a subject which should be made part of the very essence of mathematics throughout all of the courses. It should have a place of no trivial importance in the junior high school work, surely with no use of the term "functionality," likely with no reference to the words "variability" or "variation," but present in every chapter of the text either in tables of related values, as tables of squares, or tables of simple interest, as pairs of related values for graphic representation, as statistical or scientific graphs, in number pairs for simple formulas, and in geometrical diagrams exhibiting comparison or relationship. The subject of functionality or variation should recur again and again, finding new importance, uses and applications, growing in emphasis through the courses in

mathematics and the sciences, taking on the dignity of a name. For emphasis and unity the subject might be treated exhaustively in a separate chapter of one of the senior high school texts, but never if such treatment brings about a neglect of application and use throughout the courses.

Until one focuses his attention upon the subject of relationship or functionality he does not realize its ramifications in the fields of mathematics, science, economics, finance, life itself. The premium on life insurance or annuity at retirement is a variable or function of many elements. Parcel post rates, the amount of one's house rent or taxes, the price of one's suit or automobile, the amount of one's income tax are all relations dependent upon many other variable relations.

We say that something does not "function" properly. We imply that it does not cause to be brought about the expected relations, correspondence or dependence. The anticipated variable result is lacking.

Variability or functionality in high school mathematics, is, as it were, one of the themes of the symphony coming up again and again, always in a new guise equally interesting, equally novel, related to the former version in the different key, familiar yet almost unique, often coming upon one unawares. When one has browsed several years in secondary school mathematics, he finds himself awaiting it, expecting it, always with pleasant anticipation, for more than likely the key and the time will be delightful altogether beyond his expectation. Moreover it is a theme which, because of its novelty, appeal, interest and variety, becomes a part of the student if he is a sympathetic subject and remains so, long after the symphony as a whole is a memory—a theme recurring from time to time, almost as uniquely one's own as if he had discovered and recorded it originally.

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A MATHEMATICAL NIGHTMARE [1]

By JOSEPHINE SKERRETT

Class of 1930 Simon Gratz Senior High School, Philadelphia, Pa.

Prologue

As a member of the Mathematics Club, I do not come before you to solicit your membership. Instead, it is my purpose and intent to prepare your minds for the little sketch which will follow me.

No doubt you are wondering what kind of a play would be presented by a mathematical group. Probably you suspect something "deep," with the characters using highly technical language; or perhaps you expect something fantastic, with the characters dressed as triangles or squares and performing a formal mathematical dance.

Whatever your thoughts on the subject may be, you are probably mistaken. Mathematics is a live subject and our little sketch is a picture taken from life. The story is of a high school boy, a very unusual boy—would you believe it, he did not like mathematics! and—well you shall see for yourselves.

The Play

Characters

Bob Weston, a high school sophomore.
Earl Weston, an engineer, about twenty-three years of age.
Jean Weston, a bookkeeper, about twenty years of age.
Mrs. John Weston.
Mr. John Weston.
Fred Frazer, Bob's school chum.
Court Crier, from Caesar's court.
Two Soldiers.
Knight.

[1] The members of the Mathematics Club of the Simon Gratz Senior High School in Philadelphia were asked by their sponsor, Mr. Milton Brooks, to submit original plays for presentation at a school assembly. This is the play selected.

Act I

Place—Living room of the Weston home.

Time—Evening.

Bob is seated at desk, reading.

Bob (closing book and rising): Well, that's done. I'm positively fed up on Caesar. You get Caesar in Latin, and Caesar in English, and Caesar in history. I'll be dreaming about Caesar next—Now what?—(Goes to desk and picks up book—groaning) Algebra! (Sits down—reading) Page one hundred, problem four. (Reading) "Mary's age plus Ann's age equals forty-four years. Mary is twice as old as Ann was, when Mary was half as old as Ann will be, when Ann will be three times as old as Mary was, when Mary was three times as old as Ann. How old are Mary and Ann?"—Whee—what a problem! Let's see. Mary is twice as old as Ann was when—oh, I'll do it tomorrow.—Gee, but I'm sleepy. (Falls asleep.)

Curtain

Act II

Place—Living-room of the Weston home.

Time—Morning.

Mrs. Weston is arranging some articles on the table.

Enter Court Crier followed by soldiers.

C. C.: Hear ye! Hear ye! (reading from scroll) "The almighty Caesar hath decreed that on this the .....day of ..... nineteen hundred and twenty-nine, and henceforth, the study, use, or persuance, in any shape or form, of the subject of mathematics is prohibited. Anyone caught violating this law, shall be condemned to instant death."

Soldiers, seize all mathematics books, newspapers, calendars, clocks, and other mathematical devices.

(The command is carried out and they depart.)

Meanwhile Bob has entered.

Mrs. W.: Bob, why are you home from school so soon?

Bob: No mathematics, no school. They couldn't make out our rosters.

Mrs. W.: My goodness, school hasn't stopped forever?

Bob: No such luck! We have to be back in two weeks. The rosters may be made out by then. Besides, now they can't teach

Algebra, or Geometry, or Trigonometry, or Commercial Arithmetic, or Bookkeeping, or Mechanical Drawing, and they have to revise the course in Physics and Chemistry. (Enter Earl) Hello Earl, aren't you home rather early?

(Earl sits down dejectedly.)

Mrs. W.: What is the matter, Earl?

Earl: No work. Engineering involves so much mathematics that it's worth your life to carry out operations. I don't know what I'll do now.

Bob: Tom Richards is awfully mad because he wanted to be a surveyor and now he can't. And Fred said that Astronomy will be impossible now, without mathematics.

Earl: Looks as if we'll all have to take up farming.

Bob: It might even be dangerous to do that because farming now-a-days is so scientific.

Earl: But the most important thing is what to do now.

Mrs. W. (perturbed): Oh, I don't know what to say—Why don't you two go to the theater and cheer up?

Bob: That's just it. All of the Movies are closed. They have to use mathematics in some way, I don't know how. And there's only one good show in town and I've seen that.

Earl: And the lecture at Town Hall this evening on Modern Finance had to be cancelled—because it would have used considerable mathematics and the lecturer was afraid to risk his life.

Mrs. W.: Well, go to the library.

Bob: That's closed, too. They have to use mathematics to keep the books in order.

Mrs. W.: Can't you call up some of the boys and go out?

Bob: The phone is disconnected—more mathematics. (Enter Jean.) Here's Jean.

Mrs W.: Jean! Don't you have any work either?

Jean: No, Mother. All the bookkeepers were just informed about Caesar's law. Bookkeeping can't possibly be done without mathematics. (Disgustedly) So I guess I'll stay home and wash dishes.

Earl: That'll be a good way to get rid of the dishes.

Mrs. W.: Can't you get a position as typist or stenographer?

Jean: No. I did try at three places, but all the positions were filled.—Doris is out of work, too. She is an accountant.

(Enter Mr. Weston.)

Bob: Here comes Dad. Looks like a family reunion.

Mrs. W.: Why John! What is wrong?

Mr. W.: I'm out of work. The firm closed down to-day.

Mrs. W.: Out of work!

Mr. W.: How can a brokerage exist without mathematics? All the companies called in their stock to-day, and now I have nothing to do. (Takes newspaper from pocket, glancing right and left.)

Earl: Say, Dad, where did you get the newspaper?

Mr. W.: Sh-h. From my golf caddy. He's the newsboy now. (Looking at paper.) It says here that thousands of people are out of work.—The stock exchange has collapsed; all banks are closed; all architects and draftsmen are out of work; building construction has ceased; several factories and mills have shut down.

Jean: How long do you think this wretched law will be in force?

Mr. W.: Until the people overthrow it.

Earl: Oh for another Brutus!

Bob (thoughtfully): There is only one thing to be thankful for.

Earl: What's that?

Bob: We won't be pestered with insurance agents any more.

Jean: I can't understand why you insist upon joking at a time like this. All of us are out of work and there is no telling when we will be back at work again.

Earl: Joking! Who's joking? I'm serious. Do you realize I was to be married in two weeks and now that is impossible, since I'm out of a job? By the way, Mother, what time is it? I'm hungry.

Mrs. W.: I don't know. The soldiers came and confiscated all the clocks. Hereafter, you will have to rely upon the sun.

Bob (adding): And upon your stomach.

Mr. W.: I wonder how long we shall have food to satisfy it.—Oh, Caesar! Why did you bring this upon us?

(Enter Knight.)

Knight: Ho—neighbor! Assist! Death to the tyrant Caesar! Civilization is dependent upon mathematics and mathematics we will have. To death with Caesar!

All: To death with Caesar!

Curtain

Act III

Place—A street.

Time—Morning.

Enter Bob from right, Fred from left.

Bob: Hello, Fred.

Fred: Hello, Bob. Did you do that problem?

Bob: Do you mean about Mary and Ann?—No. I'll do it later. I had a peculiar dream last night and I've come to the conclusion that mathematics is a necessary evil.

Fred: Evil! Why mathematics is good. It's full of thrills. Lots of people enjoy doing it. Your brother and sister like it.

Bob: Yes, but I don't see why.

Fred: Come to the Math Club some afternoon and we'll show you why. It's lots of fun.

Bob: All right. But do you realize that it is exactly three minutes of nine?

Fred: Three minutes of!

(They start to hurry off stage.)

Bob (stopping): Wait a minute. How old were Mary and Ann, anyway?

Fred: Come around to Math Club this afternoon and find out.

Bob (looking at watch): Two minutes of nine.

(Exeunt hurriedly.)

Curtain

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A PLAN FOR MEETINGS OF MATHEMATICS TEACHERS IN A HIGH SCHOOL

By H. P. McLAUGHLIN

Head of Mathematics Department, Dorchester High School for Boys, Dorchester, Massachusetts

The Problem

The writer is the head of a mathematics department containing six teachers besides himself in a high school enrolling about 1800 boys each year. The principal finds that his administrative duties keep him from giving as much time to supervision as he would like to give so that he has asked the heads of the different departments to hold meetings of their departments to assist him in improving the instruction in the school. This paper will be confined to a plan for teachers' meetings of this kind.

The writer's experience in holding such meetings during the past school year has raised the following questions:

1. At what time of the day should they be held?
2. How often should they be held?
3. Where should they be held?
4. How long should they be?
5. What should be done at these meetings?
6. What kind of notice should be given to teachers concerning the meetings before they are held?
7. How should they be conducted?

These questions will be discussed in this paper in the order in which they are mentioned. The first question has been answered by various principals according to the division of the day in their own schools. In high schools there is usually only one session, so that meetings must come at the close of the day or not at all. The question becomes then: how soon after dismissal should they be held? Most teachers would probably agree in saying "As soon as possible." It is necessary to set the time definitely, however, to prevent straggling. Five minutes after the dismissal bell has been found to be a sufficient allowance of time for going to the place of meeting.

When the number of meetings is being decided on, the total

number of teachers' meetings held in the school is an important consideration. There is a fairly general agreement amongst educators that school meetings for teachers need not be held oftener than once in two weeks unless some unusual situation is encountered. To avoid conflicts, and to secure a well balanced schedule of meetings, it is advisable that the principal and the heads of departments plan their schedules in a special conference at the beginning of the year. The writer has found that one meeting of his department each month has apparently satisfied the needs of the teachers under his supervision.

The place of teachers' meetings is usually determined by the number of teachers, as in most cases there is only one room in the building that will accommodate all of them, but the number of teachers in a high school department is usually small enough to make round table conferences possible. T. T. Allen argues very convincingly for such meetings in his article entitled "Teachers' Meetings on a Democratic Basis." He claims that teachers should face each other and not the leader in teachers' meetings. Certainly we cannot expect teachers to be instrumental in developing a love for democracy in their pupils, when the organization of school meetings discourages it in the teachers themselves. Most schools are fortunate enough to have at least one table and sufficient loose chairs so that departmental meetings at least may escape the curse of the strait-jacket position imposed by the ordinary high school desk and seat.

There are times, however, when it is helpful to have the meeting in a classroom. For example the leader of the meeting may wish to show how to teach the use of the slide rule. In his presentation he must show where the demonstration slide rule is to be hung in the classroom. The visibility of the materials used from all parts of the classroom is of course an important consideration in every demonstration lesson. A whole meeting might very well be given over to the topic "What can we do for the pupils who must sit in the furthest corners of a crowded classroom?" with teachers posted in these corners to report on how much they can see of what is being put on the front blackboard.

The available times and places of meeting having been determined, the next question to be decided is: how long should they be? As an hour is considered long enough for most academic meetings held in the morning, it would seem unwise, if not use-

less, to hold them during longer periods for teachers who are more or less fatigued by the day's work. In this connection, it is well to remember that teachers resent a meeting which is long drawn out to such an extent that they are in no frame of mind to profit by anything that is said after the first hour. Writers who have expressed themselves on the subject, feel that one hour is long enough for nearly all school meetings.

The time for closing the meetings should be definitely understood, so that teachers may feel free to make other appointments late in the afternoon. Teachers have a wide variety of interests and wish naturally to make every hour count for something. Furthermore the closing time should be rigidly observed. If some of the teachers desire to remain so that they may continue the discussion in informal groups so much the better, but the leader of the meeting should make an explicit announcement at the appointed time, that any teacher who has some other appointment should feel free to leave. Consideration for anyone summoned to a meeting is almost a guarantee of his cooperation.

How this hour may be used to the best advantage depends, of course, on the needs of the teachers. Some of them should be used to stimulate professional growth. Others should deal with the perennial problems of every school. Others again should deal with the special problems of the department. Parts of some meetings should deal with what the supervisor sees in the classrooms. Most experienced teachers teach some one topic remarkably well. They should be asked to tell the other teachers just how they do it. An alert supervisor will try to inform himself concerning the talents and experience of all of his teachers. He may find that one has had practice at sea in applying the mathematics of navigation. Another has been a surveyor during the summer time. A third has worked in an architect's office, and so on. Such teachers may easily be induced to work up mathematical projects and to describe them for their fellow teachers.

In the meetings held by the writer during the last year, one teacher described his experience as a corrector of College Entrance Board examinations. This talk was especially valuable in showing what the examiners looked for. Another teacher gave a talk on some research work he was doing in mathematics in connection with a course he was taking. This proved so interesting that not a teacher left at the end of the scheduled meeting, although all of them knew they were free to do so.

Even the various seasons have their special problems. On the day before the opening of school in September, the writer urges the teachers not to wait until the second day of school to begin a lesson and not to wait for boys who are still away on their vacations, but to start teaching the very first day of school. In January, teachers are reminded that parents of pupils, who are failing, should have a special warning sent to them through the mail. Mimeographed forms for this purpose are distributed at the same time. The warning is sent in January because the winter has proved in the past to be a much better time for improving a mark than the spring. After the April vacation, the reminder deals with the necessity for allowing for "spring fever" in boys and the necessity for making a study of out-of-door projects; for example, the setting-up of batter boards for the foundation of a house with the help of a transit.

Several writers advocate that topics to be discussed at teachers' meetings during the year should be listed and that a mimeographed notice of these topics with the dates on which they are to be discussed, should be given to all the teachers. Beyond the shadow of a doubt, it would be of great help to a head of department to make such a calendar, but the value of a mimeographed copy to the other teachers requires further demonstration. The discussion during the very first meeting often shows the necessity of modifying the original plan. Another consideration is the deluge of printed and mimeographed material that assails the teacher in a large city high school.

For the year 1929–1930, the writer has worked out for his own guidance the following plan of readings and of meetings, and the topics to be discussed as soon as routine matters and the special problems of the season have been considered.

General References

Teaching Junior High School Mathematics—H. C. Barber, Houghton, Mifflin Co., Boston.

A Diagnostic Study of the Teaching Problems in High School Mathematics—W. D. Reeve, Ginn & Co., Boston.

Scientific Method in the Reconstruction of Ninth-Grade Mathematics—H. O. Rugg and John R. Clark, University of Chicago Press.

The Teaching of Junior High School Mathematics—D. E. Smith and W. D. Reeve, Ginn & Co.

The Teaching of Mathematics in the Elementary and the Secondary School—J. W. A. Young, Longmans, Green & Co.

The Teaching of Mathematics in Preparatory Schools. Report of the Boys' School Committee of the Mathematical Association—G. Bell & Sons, London.

A Brief List of Mathematical Books Suitable for Libraries in High Schools and Normal Schools, New York City. Bureau of Publications, Teachers College, Columbia University.

SEPTEMBER MEETING

Motivating the Study of Mathematics

References:

"Objectives in the Teaching of Mathematics"—W. D. Reeve, MATHEMATICS TEACHER, Nov., 1925.

"A Project in Mathematics. Two Algebra Classes Build a Railroad"—D. P. Smith, MATHEMATICS TEACHER, Feb., 1925.

"Geometry Detected by Sherlock Holmes"—B. B. Hedges, MATHEMATICS TEACHER, March, 1921.

"The Human Significance of Mathematics"—W. L. Schaaf, MATHEMATICS TEACHER, Dec., 1926.

"The Call of Mathematics"—D. E. Smith, MATHEMATICS TEACHER, May, 1926.

"The Formula in Ninth-Grade Algebra"—Dr. J. M. Kinney, MATHEMATICS TEACHER, Nov., 1921.

Problem Exercises for High School Teachers—D. Waples and Others, University of Chicago Press.

OCTOBER MEETING

Teaching the Language of Mathematics

References:

"Teaching the Algebraic Language to Junior High School Pupils"—J. R. Overman, MATHEMATICS TEACHER, April, 1923.

"The Teaching of Beginning Geometry"—A. J. Schwartz, MATHEMATICS TEACHER, May, 1922.

Problem Exercises for High School Teachers—D. Waples and Others, University of Chicago Press.

NOVEMBER MEETING

Laboratory Methods of Teaching Mathematics

References:

"A Laboratory Method of Teaching Mathematics in the Classroom"—Charles A. Stone, MATHEMATICS TEACHER, April, 1924.

"The Laboratory Method in the Teaching of Geometry"—C. A. Austin, MATHEMATICS TEACHER, May, 1927.

"Experimental Geometry"—G. A. Harper, MATHEMATICS TEACHER, March, 1922.

DECEMBER MEETING

How Can a Pupil be Taught to Prove a Theorem in Plane Geometry

References:

"Adapting Plane Geometry to Pupils of Limited Ability"—M. Hillebrandt, MATHEMATICS TEACHER, February, 1925.

"When is a Proof Not a Proof"—P. Stroup, MATHEMATICS TEACHER, December, 1926.

JANUARY MEETING

The Assignment of Home Lessons

References:

"Classroom Methods in Teaching Algebra"—J. A. Nyberg, MATHEMATICS TEACHER, October, 1926.

"The Problem of Algebra Instruction"—J. J. Birch, MATHEMATICS TEACHER, March, 1927, pp. 168–169.

"Teaching Pupils How to Study"—A. Davis, MATHEMATICS TEACHER, October, November, and December, 1921.

Problem Exercises for High School Teachers—D. Waples and Others, University of Chicago Press.

FEBRUARY MEETING

Examinations Their Uses and Abuses

References:

"What the Tests Do Not Test"—H. M. Walker, MATHEMATICS TEACHER, January, 1925.

The Present Status of Written Examinations and Suggestions for their Improvement—University of Illinois Bulletin, No. 17.

An Introduction to the Theory of Education Measurements—W. S. Monroe, Houghton, Mifflin Company.

A Test in Plane Geometry—R. S. Schorling, Lincoln School, Teachers College, N. Y.

"A New Type of Final Geometry Examination"—Vera Sanford, MATHEMATICS TEACHER, January, 1925.

Instructional Tests in Algebra—Schorling, Clark, and Lindell, World Book Co., Chicago.

"An Achievement Test in Solid Geometry"—L. A. McCoy, MATHEMATICS TEACHER, March, 1928.

Problem Exercises for High School Teachers—D. Waples and Others, University of Chicago Press.

"Educational Tests—To Standardize or Not to Standardize"—W. D. Reeve, MATHEMATICS TEACHER, November, 1928.

MARCH MEETING

How to Make a Self-Rating Scale for Mathematics Teachers

Part I—The Classroom

References:

The Supervision of Instruction—Barr and Burton—Chapters 4, 12.

"The Supervisor at Work"—Anderson and Bush, *Journal of Educational Method*, December, 1923, January, February, March, 1924.

"Rating of Teachers"—Carrigan, *Journal of Educational Method*, October, 1922.

"Executive Control by Means of a New Type of Measurement"—Herring, *Journal of Educational Method*, November, 1924.

Administration and Supervision of the High School—Johnson, Chapter 18.

"Teacher Rating in the United States"—*Elementary School Journal*, January, 1928—pp. 327–331.

"Judging and Rating the Teachers"—Gilmore, *Educational Review*, December, 1927.

Journal of Educational Research, May, 1920—Article by Connor.

School Supervision in Theory and Practice—Collings—Chapter 11, especially.

The Question as a Measure of Efficiency—Romiett Stevens, Bureau of Publications, Teachers College.

Class Procedure Tests—Waples and Others, University of Chicago Press.

"A Technique Used in Training Principals to Supervise Instruction"—Smith, *Elementary School Journal*, February, 1928.

"Some Problems in Scientific Study of Classroom Supervision"—Barr, *Journal of Educational Method*, November, 1927.

"Can High School Supervision Be Made Scientific?"—C. H. Judd, *School and Society*, March 24, 1928.

"Educational Supervision"—First Yearbook of National Conference on Educational Method, Section 3.

APRIL MEETING

How to Make a Self-Rating Scale for Mathematics Teachers

Part II—Preparation for the Lesson, Use of Models, etc.

References: As above.

MAY MEETING

How to Make a Self-Rating Scale for Mathematics Teachers

Part III—The Lesson Itself

References: As above.

JUNE MEETING

How to Make a Self-Rating Scale for Mathematics Teachers

Part IV—Results in Terms of Pupil Reactions

References: As above.

The following scheme of notifying teachers of the times and places of the meetings has been used with satisfactory results. After the writer had consulted with the members of his department, he found that a meeting on the third Wednesday of the month would not conflict with any other periodical engagement that the teachers had already made. Then having settled on this as the time of meeting, he placed a reminder in each teacher's box in the office the day before each meeting was actually held. The result has been that no teacher has been absent from any meeting unless he had received some special assignment from the head master, which could not have been foreseen. In such cases, the teacher has never failed to state in advance why he must be absent.

The organization of the activities in the meetings of a small group of teachers such as a department is a simple matter. The head of the department is himself teaching the same kind of classes as the other teachers, and is subject himself to visitation by the principal and superintendent so that he is usually in close touch with the problems and trials of the members of his department. His position is similar to that of a captain of a football team. He is not necessarily the ablest of the group by innate capacity but he ought to be more experienced, and better trained. His daily classroom experience tends to keep his feet firmly on the ground all of the time, and this fact usually makes itself felt in his conversations with his co-workers.

A round table organization for the meetings he leads has already been suggested. The machinery of departmental meetings will naturally be very simple. Committees of one member each will be common. Formalities of all kinds may be dispensed with. The speaker of the meeting need not rise unless he has board work to do or a similar performance which requires a standing position. Every detail of the meeting should be arranged so as to promote a real democracy in which the head of

the department may maintain his leadership only by being better prepared, and by rendering more sympathetic help than any other member of the group. A leadership of this kind makes each meeting not only a source of inspiration to the members of the department but it is equally valuable to the head of the department himself.

ON TO ATLANTIC CITY!

It is hoped that all members of the National Council of Teachers of Mathematics who can will attend the annual meeting of the Council at Atlantic City on February 21st and 22d, 1930.

President Barber is building the program around the topic "The Curriculum in Geometry." There will be a report by the Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics on the question of "A Combined Course in Plane and Solid Geometry for the Tenth Grade." Professor Dunham Jackson of the University of Minnesota is Chairman of this Committee.

The list of speakers will include such leaders as Professors W. R. Longley of Yale University, Oswald Veblen of Princeton University, W. R. Ransom of Tufts College, John C. Stone of the State Teachers College, Montclair, N. J., E. H. Taylor of the State Teachers College at Charleston, Ill., E. W. Schreiber of the State Teachers College at Macomb, Ill., Miss Marie Gugle, former president of the National Council of Teachers of Mathematics, Columbus, Ohio, and Mr. George W. Evans of Boston, Mass.

The chairman of the local Committee at Atlantic City is Mr. Burton W. Liddell, Atlantic City High School. Mr. Fletcher Durell is chairman of the Reception Committee.

Headquarters of the Council will be at the Chalfonte-Haddon Hall Hotel where all the meetings will be held. Reservations at this hotel or some other should be made at once.

A more detailed program of the meeting will be published in a later number of THE MATHEMATICS TEACHER.

HANDLING PROBLEMS IN ARITHMETIC BY TYPE

By CLARK D. MOORE

Principal Main St. School

Huntington, N. Y.

Since it is a lamentable fact that pupils are not able to solve the variety of problems that is expected of them it behooves us to make an effort to enable them to do this. Every test and examination that we give show startling results, and we wonder if much of our teaching is not in vain.

We are all aware that a pupil or a group of pupils may seemingly have learned perfectly the process of solving a particular type of problem. When examination or Regents day arrives the ability to solve this type of problem seems to have disappeared completely.

It is foolish for us to ask the reason. The reason is obvious. The process has not been overlearned. This may be because of a lack of time to devote to that particular type of problem, or it may be because of a lack of application on the part of the pupils. We all know that there must be frequent repetition of any activity, or at least a "doing again" at intervals, in order for a mental process to function properly when called for. This is a law of learning. In other words, we must drill and review type problems, as we may call them in the upper grades, just the same as we drill and review our fundamental processes in the lower grades.

Look back over your own school days. If you were a good student and wished to retain the ability to solve the different types of problems you had learned, either because of your own personal desire, or because examination was staring you in the face, you went through something like the following process: You began in your arithmetic text where the year's work started. You picked out the different types of problems as you went through your text and said, "I can solve this kind," or "I'm not quite sure whether or not I can solve this kind," or "I know I can't solve this kind because I couldn't work them very well when he had them." Of course most of our pupils are not like this. What an easy task the teaching of arithmetic would be if they were!

But it seems to the writer that this furnishes us the idea for, and the method of, review and drill; or, if you will, the method

we may use in helping the pupils to retain the abilities to solve the different type of problems. The teacher should make out a list of the different types of problems. Under each type she should have one or more good examples. She should make regular periodic reviews of each type. It might be done in the following manner: For one week she might take three different types of problems that had been covered; let us say a commission problem, simple interest problem and a square root problem. These problems would be kept on the board all week. These problems or others like them would be gone over each day (or as many days during the week as necessary). In short, review and drill. The next week three other types of problems would be used. The amount of time spent upon each group and the number of problems covered each week would, of course, depend upon circumstances. But such a plan, if followed consistently, could not fail to produce good results.

NEWS NOTES

The fourth annual conference of teachers of mathematics was held at the Old Capitol in Iowa City, Iowa, on October 11 and 12 under the auspices of the University of Iowa Extension Division, College of Education and Department of Mathematics Cooperating.

The program follows:

FRIDAY MORNING, OCTOBER 11, 1929

R. P. Baker, Associate Professor of Mathematics, presiding.

"Aims and Aids in the Teaching of Geometry," Beulah Shoesmith, Hyde Park High School, Chicago, Illinois.

"Some Infinities and Their Significance in Secondary Mathematics," E. W. Chittenden, Professor of Mathematics.

"Individual Drill in Junior High School Mathematics," Selma Lindell, Teacher's Training School, University of Michigan, Ann Arbor, Michigan.

Discussion

FRIDAY AFTERNOON, OCTOBER 11, 1929

Roscoe Woods, Assistant Professor of Mathematics, presiding.

"Some Teaching Values in the Higher Mathematics that a High School Teacher Should Know," J. O. Hassler, Professor of Mathematics, University of Oklahoma, Norman, Oklahoma.

"Preventive Measures in Arithmetic," Ross H. Beal, University of Iowa.

Discussion

Symposium on Topics Suggested by Those in Attendance.

FRIDAY EVENING, OCTOBER 11, 1929

Conference Dinner. W. E. Beck, Principal of the Iowa City High School, toastmaster.

Informal Symposium

SATURDAY MORNING, OCTOBER 12, 1929

J. F. Reilly, Professor of Mathematics, presiding.

"Ratio and Proportion—a Thread in High School Mathematics," Miss Lindell.

"Is Home-work an Important Factor in the Teaching of Mathematics," Miss Shoesmith.

"The Use of Project Teaching in Mathematics," Professor Hassler.

Discussion

Section 19 (Mathematics) of the New York Society for the Experimental Study of Education held its first dinner meeting of the year at the Men's Faculty Club of Columbia University on Saturday evening, October 12.

Professor Reeve, of Teachers College, Columbia University, who is chairman of the section, led the discussion on "The Universality of Mathematics."

There were 91 people present which is equal to the average number who attended the meetings last year. The section has already decided to send one or more representatives to the annual meeting of the National Council of Teachers of Mathematics to be held at Atlantic City on February 21 and 22, 1930.

The next meeting of the section will be held at the Men's Faculty Club of Columbia University on November 23 at 6:30 P.M. Professor David Eugene Smith will be the guest of the section and will lead the discussion.

The Association of Mathematics Teachers of New Jersey held its 35th regular meeting at the State Teachers College at Upper Montclair, N. J., on October 19, 1929, with F. J. Crehan of Columbia High School, South Orange, N. J., presiding.

The following program was rendered:

MORNING SESSION

"Manipulative *vs.* Associative Skills in Algebra," Professor John C. Stone, State Teachers College.

Discussion led by Dr. Frank R. MacMackin, Dickinson High School, Jersey City.

AFTERNOON SESSION

"Unit Difficulties in Solving Geometric Originals," Mr. Harrison E. Webb, Market Street High School, Newark.

Discussion led by G. W. Garthwaite.

During the Spring and Summer Session of Teachers College, Columbia University, the following members of the National Council of Teachers of Mathematics had their dissertations for the Ph.D. approved by the Committee on Higher Degrees and the examinations covering the final work of them were taken:

1. Alan R. Congdon, University of Nebraska, Lincoln, Neb. His thesis was on "Training in High School Mathematics Essential for Success in Certain College Subjects."
2. David Kasir, Mosul, Iraq, whose thesis was on "Omar Khayyam's Algebra."
3. Clarence McCormick, State Teachers College, Weatherford, Okla., whose thesis was on "The Teaching of General Mathematics in the Secondary Schools of the United States."
4. James Powell, College of the City of New York, whose thesis was on "A Study of Problem Material in High School Algebra."

These studies are all being published by the Bureau of Publications, Teachers College, Columbia University.

The following poem appeared in *Harper's Magazine* for June. The *Literary Digest* referred to it as *sublimated mathematics* whose *real theme is a snake.*

HIGHER MATHEMATICS

By Margaret Emerson Bailey

Not in slight nor in derision,
But with pencil-point precision,
He who made Pythagoras
Made the snake that parts the grass;
Made the texture of its skin
Parchment to work problems in.
He who drew an adder's spread
To a rhomboid at its head,
Let no ferrule falter loose
On the straight hypotenuse
Obliquely slanting the dull eye
Of the snake that rustles by.
Not the rod of Roman lictor
Could lay lashes any stricter
Than the stripes, exact and neat,
Parallels that will not meet,
Running straight from tip to tail
On a buckling coat of mail
Fashioned out of lozenges.
And as intricate as these—
Pointed triangle and square,
All the rigid shapes that are,
Octagon and pentagon,
He has deftly patterned on
Some unscrolled and rippling back
As a method of attack
Till every scale and single joint
Proves some problematic point,
Working out with sublety
A divine Geometry.

The Association of Teachers of Mathematics in the Middle States and Maryland will hold their 27th annual meeting at the Atlantic City High School on Saturday, November 30, 1929, beginning at 10 A.M. Mr. C. N. Shuster of the Trenton State Teachers' College will speak on "Applied Mathematics for Secondary Schools," and Mr. John Swenson of Wadleigh High School, New York City, will speak on "The Teaching of Calculus in the Senior High School."

NEW BOOKS

The Teaching Unit: A Type Study. By DOUGLAS WAPLES AND CHARLES A. STONE. D. Appleton and Company, New York, 1929. Pp. 205 + x. Price $2.00.

The purpose of the study is well set out in the beginning and again at the end. "The present volume undertakes to describe and illustrate a workable procedure for the selection of methods useful in teaching of subject matter organized in the form of units."[1] The authors choose the subject of junior high school mathematics for the study on "account of the concreteness" and "ease of analysing." While some of us wish they had chosen some other unit than positive and negative numbers, this in no way affects the purpose of their study; to quote again, from page 23, "the purpose of this study, however, has nothing whatever to do with the validation of subject matter taught but, instead, seeks to determine efficient methods of teaching subject matter." The purpose is to show a way in which a unit of subject matter may be organized for most effective teaching, using a unit in positive and negative numbers, and assuming that it will be possible to apply this same technique to other units in the same or different subjects.

[1] Page 3.

The study analyses the unit into items of knowledge and desired outcomes and then suggests ways and methods of teaching it by showing what difficulties were encountered in the classes taught, and how they were remedied. The difficulties are divided into four headings, difficulties in computation, difficulties in applying principles to life situations, difficulties in mastery of technical terms and pupils difficulties resulting from deficiencies in personal traits. Solutions of difficulties peculiar to the given unit are discussed in the next to the last chapter, the last chapter being a short general discussion on applying this technique to a given unit.

The book is well written throughout and the method of procedure is clearly described and the results well set forth in tables and summaries. The magnitude of the task performed is quite apparent, probably so apparent that only the most fortunately situated investigators will have the courage to attempt to build other units after the same plan. Without these other studies the purpose of this particular study will be largely defeated, for it is truly a type study.

The book is well worth reading and will doubtless afford much aid in the teaching of this particular unit. The teaching methods are colored somewhat by the "laboratory plan" of teaching, which was used in the experimentation, but adaptations may be easily made in most cases. It is, however, highly important that the purpose of the book be kept in mind if much of lasting value is to be gained. That is the book should be the stimulus for the working out of other like units or units somewhat like this in several different subjects. In this way the whole field of teaching subject matter may be improved instead of a very small, almost insignificant, fraction of it.

JAS. H. ZANT.
STATE TEACHERS COLLEGE,
DURANT, OKLA.

Modern Mathematics, Seventh School Year and Eighth School Year. New Edition. By RALEIGH SCHORLING AND JOHN R. CLARK. World Book Company, Yonkers-on-Hudson, New York. 1929. Pp. 274 + xiv and 306 + xiv. Price $0.88 and $0.92.

Upon a hurried first glance at the New Edition of the Schorling-Clark books one who is rather familiar with the first edition is likely to think that it is the same thing with a few pages of problems added at the end of each chapter. Upon closer inspection however one will find that the revision goes much deeper. The authors stated in the preface (p. vii, Seventh Year) that the chief of several modifications, made as the result of investigations carried on during the last five years, "deals with the most crucial issue with which school men are now struggling: "How can we gain desirable outcomes of individual instruction without unduly disturbing the machinery of a going school?"

The authors attempt to do this through "three lists of specially graded problems, for slow, average, and superior pupils, given at the end of each chapter." They state however that these can be used while the pupils are working through the chapter. The same thing is aimed at in the testing program on fundamental computation, also placed at the end of each chapter. Uniform time limits and standards are set, for three levels of ability, with provisions for advance to the next drill when the pupil has completed one

satisfactorily. "These features permit children to progress at their own rate according to their own ability, without extra work on the part of the teacher."

There seems to be no doubt that the purpose of the authors will be accomplished, at least in part by these improvements. They should go a long way toward remedying one of the fundamental defects of the first editions of these rather well worked out texts. Many teachers felt that they did not contain enough problem material, especially for the upper portion of the class. The practice tests, if used carefully and systematically, will probably go a long way in giving our junior high school pupils a reasonable ability in computing by the time they have finished the eighth grade.

There is also considerable revision and rearrangement within the various chapters themselves, as well as deletion and addition of material. This is evidently made in the belief that it will improve the method of presentation. New numbers are in many cases substituted in order to make the material more vital and more up-to-date, for instance on page 45 of the seventh year book, the number of pounds of sugar for one dollar is given from 1920 to 1928, rather than from 1915 to 1920 as in the first edition. Mistakes have been corrected also.

Another interesting change is in the manner of introducing many of the topics, instead of making a bare statement of the principles, they are introduced by saying that John or Mary did certain things, etc. This was done in many cases in the old books also. Last we might mention the illustrations; while in the main the same cuts are used, every case affected by modern styles, dress, automobile, or what not, has been changed.

JAS. H. ZANT.
STATE TEACHERS COLLEGE,
DURANT, OKLA.

Plane and Spherical Trigonometry. By J. SHIBLI. Ginn and Company, New York. 1928. Pp. 311 + xii. $1.96.

The author states his aims from three standpoints, the student, the subject matter, and the teacher. For the student the book aims "to present trigonometry in such a way as to make it interesting and understandable." To do this the student is to be given an appreciation of the value of trigonometry early through an approach that is primarily psychological, "but with due regard to the logical development." The student is also given, "a historical view of the growth of trigonometry" partly at the beginning and partly interspersed with the subject matter throughout the book.

From the standpoint of the subject matter several things are in evidence in the aims; the book "provides as thorough a course in trigonometry as is given in our best schools and colleges" and "is intended for students approaching some maturity." "The applications cover a wide field," the principles of which are explained in order that the student may not be working blindly. Exercises and problems are carefully chosen and graded approximately as to difficulty. "Several matching exercises of the modern type are included for the first time in a book on trigonometry." The topics should be taught in the order given to avoid monotony but may be rearranged if desired. Tables are carefully selected and include tables of squares

and square root. It is quite apparent then that the book is written only for college classes, and for those teachers who expect to give a rather thorough course in trigonometry. It is not a brief course in any sense, neither is it long and cumbersome.

For the teacher the book has been thoroughly tested by being taught in mimeographed form at Pennsylvania State College over a period of three years by several teachers, and it is presented as a "thoroughly teachable text," "adapted to various methods of teaching and suitable for courses of various lengths."

The above purposes are in general well carried out. The applications cover a wide field and the corresponding principles are explained in the text, but it is doubtful whether the student will always be able to follow through the explanation. In many cases even the teacher will have to be better prepared in mechanics and higher mathematics in order to fully comprehend the meaning. However, this is not strictly a criticism since teachers of the type of students for whom the book is evidently intended ought to have such preparation.

The author has shown a high degree of scholarship in presenting the subject, it is evident that he has a thorough knowledge of his subject and a clear idea of a method of teaching it. It is not necessary that the teacher who uses the book be in entire agreement with this method, but the fact that there is a definite method of teaching is kept in mind while writing the book is significant, and adaptations and omissions may be easily made in cases of disagreement.

The book is well written and apparently accurate in detail throughout. Statements and phrases of special importance as well as all rules and principles are set in italics. This should add much to the utility of the book both from the standpoint of the teacher and student.

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